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LOCAL ANTIMAGIC EDGE COLORING OF GEAR GRAPHS AND SEMI PARACHUTE GRAPHS

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ABSTRACT

Graf G merupakan pasangan himpunan titik $V(G)$ dan himpunan sisi $E(G)$ yang dinotasikan dengan $G = (V(G), E(G))$. Pewarnaan pada graf merupakan pemberian warna pada setiap titik, sisi, atau wilayah dengan syarat setiap titik, sisi atau wilayah yang bertetangga tidak dapat memiliki warna yang sama. Fungsi bijektif $f : V(G) \rightarrow \{1, 2, 3, \dots, |V(G)|\}$ disebut sebagai pewarnaan lokal sisi anti-ajaib jika untuk setiap dua sisi yang bertetangga e_1 dan e_2, memiliki bobot yang berbeda yaitu $w(e_1) \neq w(e_2)$, dengan $e = uv \in E(G)$, $w(e) = f(u) + f(v)$. Bilangan kromatik merupakan istilah dalam pewarnaan lokal anti-ajaib, yaitu jumlah minimum warna yang diperoleh dari pelabelan lokal anti-ajaib. Penelitian ini membahas tentang pewarnaan lokal sisi anti-ajaib pada Graf Gear (G_n) dan Graf Semi Parasut (SP_{2n-1}). Tujuan dari penelitian ini adalah untuk menentukan bilangan kromatik pewarnaan lokal sisi anti-ajaib $\chi lea(G)$ pada graf yang diteliti. Metode yang digunakan pada penelitian ini adalah pendeteksian pola sehingga diperoleh pola umumnya. Berdasarkan analisis diperoleh bilangan kromatik pewarnaan lokal sisi anti-ajaib pada Graf Gear (G_n) dan Graf Semi Parasut (SP_{2n-1}) beruturut-turut adalah $\chi lea(G_n) = n + 2$ dan $\chi lea(SP_{2n-1}) = n + 2$.

The graph G is a pair of sets consisting of a vertex set $V(G)$ and an edge set $E(G)$, denoted by $G = (V(G), E(G))$. Coloring a graph involves assigning colors to each vertex, edge, or region such that no adjacent vertices, edges, or regions share the same color. A bijective function $f : V(G) \rightarrow \{1, 2, 3, \dots, |V(G)|\}$ is called a local edge antimagic coloring if for any two adjacent edges e_1 and e_2, they have different weights, $w(e_1) \neq w(e_2)$, where $e = uv \in E(G)$ and $w(e) = f(u) + f(v)$. The chromatic number is the term used in the context of local antimagic coloring, referring to the minimum number of colors derived from local antimagic labeling. This research discusses the local antimagic edge coloring on the Gear Graph (G_n) and the Semi Parachute Graph (SP_{2n-1}). The aim of the research is to determine the chromatic number of local antimagic edge coloring $\chi lea(G)$ for the researched graphs. The method used in this research is pattern detection to derive the general pattern. Based on the analysis, the chromatic number of local antimagic edge coloring is obtained for the Gear Graph (G_n) and the Semi Parachute Graph (SP_{2n-1}) are $\chi lea(G_n) = n + 2$ and $\chi lea(SP_{2n-1}) = n + 2$.

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INTRODUCTION

Mathematics is the study of structure, space, patterns, and quantitative relationships. As time has progressed, mathematics has evolved into a broader discipline. Discrete mathematics is a branch of mathematics often used to solve problems in everyday life. One of the topics studied in discrete mathematics is graph theory. Graph theory was first introduced by the Swiss mathematician Leonhard Euler in 1736 with the problem of the Königsberg bridge (Daniel, 2019).

A graph G is defined as a pair of sets $(V(G), E(G))$, denoted by $G = (V(G), E(G))$. $V(G)$ is a non-empty set of vertices, and $E(G)$ is pair set of two vertex and its possibly empty set of edges. A graph can have no edges but must have at least one vertex (Munir, 2010). Over time, graph theory has continued to evolve with research in various areas of mathematics. One frequently studied topic in graph theory is graph labeling and coloring. Graph labeling is a mapping from the vertices and edges of a graph to positive integers, referred to as labels. Graph labeling is divided into three types there are vertex labeling, edge labeling, and total labeling (Parkhurst, 2014). In labeling, the term "weight" refers to the value obtained by summing the labels around an object, which could be a vertex, an edge, or a region.

Coloring a graph involves assigning colors to each vertex, edge, or region. In this process, adjacent sets of vertices, edges, or regions must not have the same color. Based on the object being colored, graph coloring is divided into three types there are vertex coloring, edge coloring, and region coloring (Waluyo et al., 2023). A key term in graph coloring is the chromatic number, which represents the minimum number of colors used in a graph (Arumugam et al., 2017).

One of the topics that combines the concepts of graph labeling and coloring is the local antimagic coloring, also known as local antimagic vertex coloring. In local antimagic vertex coloring, the edges of a graph are labeled with positive integers, and the weight of a vertex is determined by summing the labels of the edges incident to that vertex (Arumugam et al., 2017). The weight obtained is then considered the color of the graph. Local antimagic vertex coloring was first introduced by Arumugam et al. (2017) in their study of local antimagic vertex coloring on various types of graphs, including Tree Graphs, Path Graphs, Cycle Graphs, Friendship Graphs, Complete Graphs, Complete Bipartite Graphs, Ladder Graphs, and Wheel Graphs.

The next research topic regarding coloring is a local antimagic edge coloring which was first introduced by Agustin et al. (2017). Local antimagic edge coloring involves labeling all edges and then finding the weight of each edge by summing the labels of the two vertices that the edge connects. The resulting weight is referred to as the color. This study is an extension of local antimagic vertex coloring. The graphs examined in this research include Path Graphs, Cycle Graphs, Complete Graphs, Friendship Graphs, Star Graphs, Ladder Graphs, Wheel Graphs, and Prism Graphs.

Agustin et al. (2018) continued their research on local antimagic edge coloring using the comb product operation. The graph studied in this research is path comb path, path comb cycle, cycle and path, cycle and cycle, path and star, cycle and star. Agustin et al. (2018) continued their research on comb products to local super antimagic edge coloring on path comb paths, path comb cycles, and path and star.

Research on local antimagic vertex coloring continues to evolve. Local antimagic total vertex coloring is an extension of local antimagic vertex coloring studied by Putri et al. (2018) on Star Graphs, Double Star Graphs, Banana Tree Graphs, Millipede Graphs, and graphs resulting from the amalgamation operation of Star Graphs.

Nisviasari et al. (2019) continued their research on local antimagic coloring, but they conducted research on total super antimagic coloring of planar graphs. The graphs studied in this research were Jahangir graphs, wheel graphs, ladder graphs, and circular ladder graphs. Then Nisviasari et al. (2021) continued their research on shackle graphs.

Recent research on local antimagic coloring was conducted by Kaindi et al. (2023), focusing on local antimagic region coloring. In this study, the elements assigned weights or colors are the regions of a graph. The graphs examined are Ladder Graphs and Circular Three-Ladder Graphs.

Based on previous research results, the author is interested in conducting a study on local antimagic coloring, specifically local antimagic edge coloring. The graphs to be studied in this research are Gear Graphs and Semi Parachute Graphs. Gear Graphs are a generalization of Wheel Graphs by adding a vertex between each pair of adjacent vertices on an n-cycle (Gallian, 2022). Semi Parachute Graphs are a generalization of Fan Graphs, by adding an edge connecting two end vertices in the Fan Graph and adding a vertex between two adjacent vertices (Mashitah, 2013).

METHOD

In this research, the method used is the pattern detection method, which is a research approach that determines the pattern of local antimagic edge coloring in such a way that a general pattern is obtained. Subsequently, the formula for vertex labeling functions and the formula for edge weight functions in local antimagic edge coloring are established, and the chromatic number of local antimagic edge coloring is determined for the Gear Graph and the Semi Parachute Graph based on the edge weight function formulas in local anti-magic edge coloring.

The stages of local coloring research on the anti-magic side in this research are as follows:

1. Determining the graph to be studied
   the types of graphs selected are Gear Graph (G_n), Semi-Parachute Graph SP_{2n-1}.
2. Notation on the graph to be studied
   At this stage, the vertices and edges of the graph to be studied are given notation.
3. Determining vertex labeling on the graph
   Labeling begins with positive integers starting from 1 until all vertices on the graph have been labeled.
4. Calculating weights
   In this step, the weights of adjacent vertices are summed to obtain the weight for the edges on the graph.
5. Determining the function on the graph
   Next, the labeling function on the graph is determined.
6. Finding the chromatic number
   The final step is to determine the minimum color based on the weights obtained.

RESULTS AND DISCUSSION

This section discusses the research results of local anti-magic edge coloring on the Gear Graph and the Semi Parachute Graph. The graphs studied are the Gear Graph (G_n) and the Semi Parachute Graph (SP_{2n-1}). The results obtained from this research include theorems regarding the chromatic number of local anti-magic edge coloring for the graphs that have been studied.

1. The local antimagic edge coloring of Gear Graph

Definition 1 A Gear Graph is a generalization of a Wheel Graph, obtained by adding a vertex between every pair of adjacent vertices in the n-cycle. The notation for a Gear Graph is G_n with $n \geq 3$.

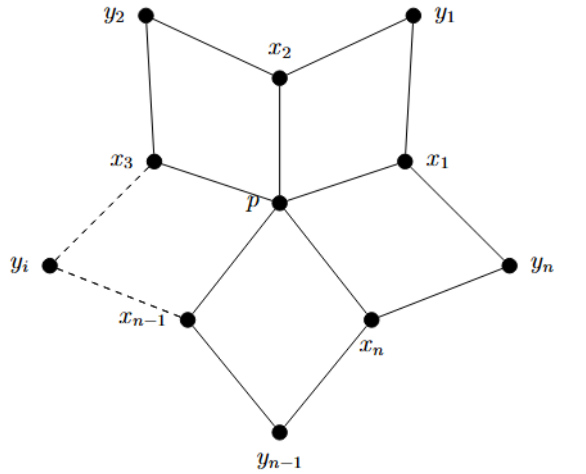

Figure 1. Gear Graph G_n

Theorem 1 Let Gear Graph G_n with n as a positive integer and $n \geq 3$, the local antimagic edge coloring of G_n is $\chi_{lea}(G_n) = n + 2$.
Proof. Given Gear Graph G_n with $n \geq 3$. Based on Definition 1 we obtain the vertex set and the edge set of the Gear Graph G_n, as follows:

$$V(G_n) = \{p\} \cup \{x_i, 1 \leq i \leq n\} \cup \{yi, 1 \leq i \leq n\}$$
and
$$E(G_n) = \{px_i; 1 \leq i \leq n\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{x_{i+1} y_i; 1 \leq i \leq n-1\} \cup \{x_1 y_n\}.$$

The Gear Graph G_n has a vertex cardinality of $|V(G_n)| = 2n + 1$ and an edge cardinality of $|E(G_n)| = 3n$. A bijective labeling function of the Gear Graph G_n is defined as follows:

$$f\big(V(G_n)\big) \to \{1, 2, 3, \ldots, 2n + 1\}.$$

The first step is to determine the labels for the set of vertices in the Gear Graph G_n. The set of vertices in the Gear Graph G_n is labeled with positive integers $\{1, 2, \ldots, 2n + 1\}$. First, the set of vertices $\{x_i\}$ will be labeled. The set of vertices $\{x_i\}$ is labeled with the labels $\{1, 2, 3, \ldots, n\}$, so we obtain:

$$f(x_i) = i.$$

Next, the label for vertex p will be determined. Vertex p is the central vertex, so it is labeled as follows:

$$f(p) = n + 1.$$

The set of vertices $\{y_i\}$ is labeled with the labels $\{n + 2, n + 3, \ldots, 2n\}$. Since the index i for the set of vertices $\{y_i\}$ ranges from $\{1, 2, \ldots, n - 1\}$, we obtain:

$$f(y_i) = 2n + 1 - i.$$

Vertex y_n is labeled with the label $2n + 1$. Thus, vertex y_n will be labeled as follows:

$$f(y_n) = 2n + 1.$$

After the vertex labeling function is obtained, the vertex labeling function can be written as follows:

$$f(v) = \begin{cases} i, & \text{for } v = x_i;\ 1 \le i \le n \\ n+1, & \text{for } v = p \\ 2n+1, & \text{for } v = y_n \\ 2n+1-i, & \text{for } v = y_i;\ 1 \le i \le n-1 \end{cases}$$

Step two is to determine the edge labeling weights on the Gear Graph G_n with $n \ge 3$. The edge labeling weights on the Gear Graph G_n are obtained by summing the labels of two adjacent vertices. The weight of five set of edge $\{ px_i \mid \text{for } 1 \le i \le n\}$, $\{x_i y_i \text{ for } 1 \le i \le n-1\}$, $\{x_{i+1} y_i \text{ for } 1 \le i \le n-1\}$, $x_n y_n$, and $x_1 y_n$ need to be determined. First, the weights for the set of edges $\{px_i\}$ will be determined by summing the values of $(f(p)$ and $f(x_i)$, resulting in the following formula for the weight of the set of edges $\{px_i\}$:

$$\begin{aligned} w(px_i) &= f(x_i) + f(p) \\ &= (i) + (n\ + 1) \\ &= n + 1 + i. \end{aligned}$$

Next, the formula for the weight of the set of edges $\{x_i y_i\}$ is :

$$\begin{aligned} w(x_i y_i) &= f(x_i) + f(y_i) \\ &= (i) + (2n + 1 - i) \\ &= 2n + 1. \end{aligned}$$

The weight formula for the set of edges $\{x_{i+1} y_i\}$ is:

$$\begin{aligned} w(x_{i+1} y_i) &= f(x_{i+1}) + f(y_i) \\ &= (i + 1) + (2n + 1 - i) \\ &= 2n + 2. \end{aligned}$$

The weight of the edge $x_n y_n$ is:

$$\begin{aligned} w(x_n y_n) &= f(x_n) + f(y_n) \\ &= (n) + (2n + 1) \\ &= 3n + 1. \end{aligned}$$

And the weight of the edge $x_1 y_n$ is:

$$\begin{aligned} w(x_1 y_n) &= f(x_1) + f(y_n) \\ &= (1) + (2n + 1) \\ &= 2n + 1. \end{aligned}$$

After the weight of edges formulas have been obtained, the edge weights can be written as follows:

$$w(e) = \begin{cases} n+1+i, & \text{for } e = px_i;\ 1 \le i \le n \\ 2n+1, & \text{for } e = x_i y_i;\ 1 \le i \le n-1 \\ 2n+2, & \text{for } e = x_{i+1} y_i;\ 1 \le i \le n-1 \\ 3n+1, & \text{for } e = x_n y_n \\ 2n+2, & \text{for } e = x_2 y_n \end{cases}$$

Based on the weight $w(e)$, it can be seen that there are five edge weights: $n+1+i,\ 2n+1,\ 2n+2, 3n+1$, and $2n+2$. However, the weight for the set of edges $\{x_i y_i\}$ is the same as the weight for the set of edges $\{px_i\}$ when $i = n$, so this weight is counted as 1. Similarly, the weight for the set of edges $\{x_{i+1} y_i\}$ and the weight for the edge $x_1 y_n$ are the same, so these weights are also counted as 1. Thus, the number of weights for the set of edges $\{px_i\} = n$, the number of weights for the set of edges $\{x_{i+1} y_i\}$ and the weight of the edge $x_1 y_n = 1$, and the number of weights for the edge $x_n y_n = 1$. Therefore, based on the number of weights of the edge set of the Gear Graph, the chromatic number of the anti-magic local edge coloring of the Gear Graph is $\chi_{lea}(G_n) = n + 2$.

Q.E.D.

Based on Theorem 1, an illustration of the antimagic local edge coloring of the Gear Graph G_n will be provided. The local antimagic edge coloring of Gear Graph G_n can be seen in Figure 2.

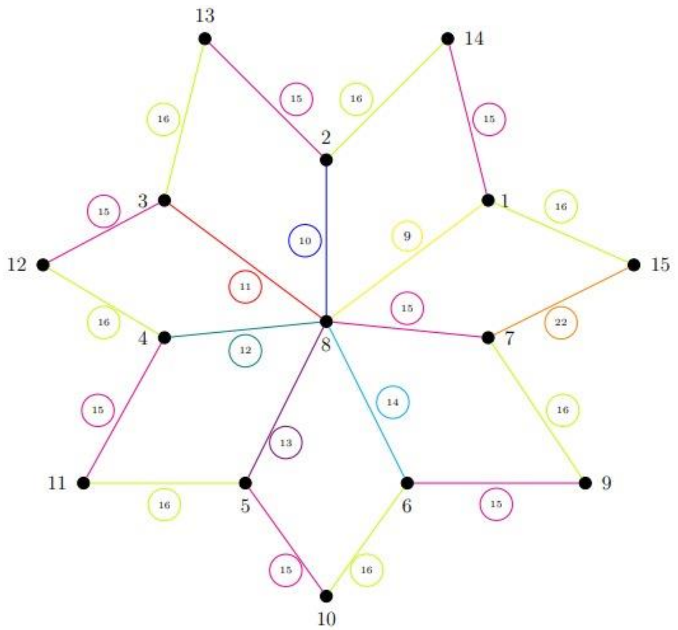

Figure 2. Local antimagic edge coloring of the gear graph $\boldsymbol{G_7}$

2. The local antimagic edge coloring of Semi Parachute Graph

Defintion 2 The Semi Parachute Graph is a generalization of the Fan Graph, obtained by adding an edge connecting the two end vertices of the Fan Graph and inserting a vertex between two adjacent vertices, except at the center vertex. The notation for the Semi Parachute Graph is (SP_{2n-1}).

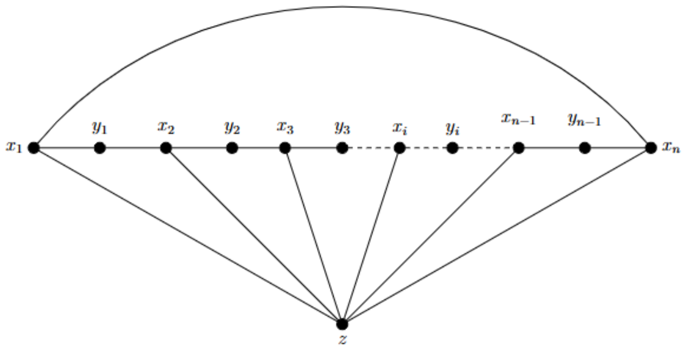

Figure 3. Semi Parachute Graph ($\boldsymbol{SP_{2n-1}}$)

Theorem 2 Let Semi Parachute Graph (SP_{2n-1}), where n is a positive integer and $(n \geq 2)$, the local antimagic edge colorng of SP_{2n-1} is $\chi_{lea}(SP_{2n-1}) = n + 2$.
Proof. Given Semi Parachute Graph SP_{2n-1} with $n \geq 2$. Based on the Definition 2, the set of vertices and the set of edges in the Semi Parachute Graph (SP_{2n-1}) as follows:

$$V(SP_{2n-1}) = \{z\} \cup \{x_i, 1 \leq i \leq n\} \cup \{y_i, 1 \leq i \leq n-1\}$$
and
$$E(SP_{2n-1}) = \{zx_i;\ 1 \leq i \leq n\} \cup \{x_1x_n\} \cup \{x_iy_i;\ 1 \leq i \leq n-1\} \cup \{x_{i+1}y_i;\ 1 \leq i \leq n-1\}.$$

The Semi Parachute Graph (SP_{2n-1}) has a vertex cardinality of $|V(SP_{2n-1})| = 2n$ and an edge cardinality of $|E(SP_{2n-1})| = 3n - 1$. A bijective labeling function for the Semi Parachute Graph (SP_{2n-1}) is defined as follows:

$$f\big(V(SP_{2n-1})\big) \to \{1, 2, 3, \ldots, 2n\}.$$

In the first step, the label set for the vertex set of the Semi Parachute Graph (SP_{2n-1}) will be determined. The vertex set of the Semi Parachute Graph (SP_{2n-1}) is labeled with positive integers $\{1, 2, 3, \ldots, 2n\}$. The vertex set $\{x_i\}$ is labeled with the set $\{1, 2, 3, \ldots, n\}$, resulting in the following:

$$f(x_i) = i.$$

Next, the label for vertex z will be determined. Vertex z is the central vertex in the Semi Parachute Graph, so vertex z will be labeled as follows:

$$f(z) = n + 1.$$

The set of vertices $\{y_i\}$ is labeled with labels $\{n + 2, n + 3, \ldots, 2n\}$. Since the index i in the set of vertices $\{y_i\}$ ranges from $\{1, 2, \ldots, n - 1\}$, it follows that,"

$$f(x_i) = 2n + 1 - i.$$

After the vertex labeling function for each vertex is obtained, the vertex labeling function can be written as follows:

$$f(v) = \begin{cases} i, & \text{for } v = x_i;\ 1 \leq i \leq n \\ n+1, & \text{for } v = z \\ 2n+1-i, & \text{for } v = y_i;\ 1 \leq i \leq n-1 \end{cases}$$

The second step is to determine the edge labeling weights for the Semi Parachute Graph SP_{2n-1} with n positive integers where $n \geq\ 2)$. The edge labeling weights for the Semi Parachute Graph SP_{2n-1} are obtained by summing the labels of two adjacent vertices. The weights of four set of edge$\{zx_i \mid \text{for } 1 \leq\ i \leq\ n\}$, $\{x_i y_i \mid \text{for } 1 \leq\ i \leq\ n\ -\ 1\}$, $\{x_{i+1} y_i \mid \text{for } 1 \leq\ i \leq\ n\ -\ 1\}$, and $(x_1 x_n)$ need to be determined. First, we will determine the weight of the edge set $\{zx_i\}$ by summing the labels of vertices $f(z)$ and $f(x_i)$. The formula for the weight of the edge set $\{zx_i\}$ is as follows:

$$\begin{aligned} w(zx_i) &= f(xi) + f(z) \\ &= (i) + (n\ +\ 1) \\ &= n + 1 + i. \end{aligned}$$

Next, the weight of the edge $x_1 x_n$ is:

$$\begin{aligned} w(x_1 x_n) &= f(x_1) + f(x_n) \\ &= (1) + (n) \\ &= n + 1. \end{aligned}$$

The weight of the edge set $\{x_i y_i\}$ is:

$$\begin{aligned} w(x_i y_i) &= f(x_i)\ +\ f(y_i) \\ &= (i)\ +\ (2n + 1 - i) \\ &= 2n + 1. \end{aligned}$$

And the weight of the edge set $\{x_{i+1} y_i\}$ is calculated as follows:

$$\begin{aligned} w(x_{i\ +1} y_i) &= f(x_{i+1}) + f(y_i) \\ &= (i + 1) + (2n +\ 1 - i) \\ &= 2n + 2. \end{aligned}$$

After the weight of edges formulas have been obtained, the formulas for the edge weights can be written as follows:

$$w(e) = \begin{cases} n + 1 + i, & \text{for } e = zx_i;\ 1 \leq i \leq n \\ 2n + 1, & \text{for } e = x_i y_i;\ 1 \leq i \leq n-1 \\ 2n + 2, & \text{for } e = x_{i+1} y_i;\ 1 \leq i \leq n-1 \\ n + 1, & \text{for } e = x_m y_m \end{cases}$$

Based on the weight $w(e)$, it can be observed that there are four edge weights: $(n\ +\ 1\ +\ i)$, $(2n\ +\ 2)$, $(2n\ +\ 1)$, and $(n\ +\ 1)$. However, the weight of the edge $(x_i y_i)$ is the same as the weight of the edge (zx_i) when $i = n$, so this weight is counted only once. Thus, the number of weights for the edge set $\{zx_i\} = n$, the number of weights for the edge set $\{x_{i+1} y_i\} = 1$, and the number of weights for the edge $(x_1 y_n) = 1$. Therefore, based on the number of weights of the edge set of the Semi Parachute Graph, the chromatic number of local anti-magic edge coloring for the Semi Parachute Graph is $\chi_{lea}(SP_{2n-1})\ =\ n\ +\ 2$.

Q.E.D.

Based on Theorem 2, an illustration of the antimagic local edge coloring of the Semi Parachute Graph SP_{2n-1} will be provided. The local antimagic edge coloring of Semi Parachute Graph SP_9 can be seen in Figure 4.

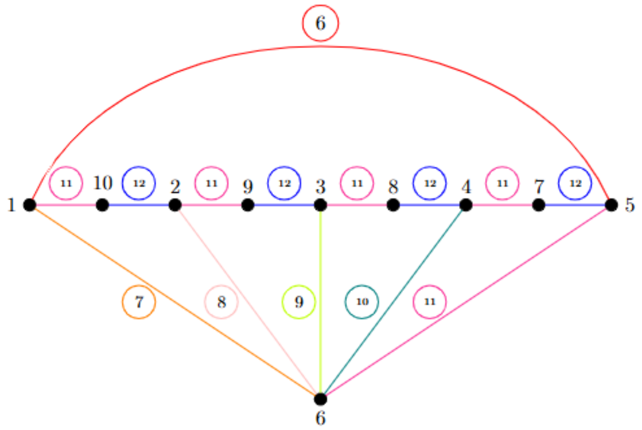

Figure 4 Local antimagic edge coloring of the Semi Parachute Graph $\mathbf{SP_9}$

CONCLUSION

Based on the results and discussion, it can be concluded that the chromatic number of local antimagic edge coloring of Gear Graph G_n is $\chi_{lea}\ (G_n)\ =\ n\ +\ 2$ and the chromatic number of local antimagic edge coloring of Semi Parachute Graph (SP_{2n-1}) is $\chi_{lea}(SP_{2n-1})\ =\ n\ +\ 2.$

An interesting direction for future work is to investigate the chromatic number under local antimagic edge coloring for Parachute Graphs and Jahangir Graphs, since this problem has not yet been resolved.

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The authors declare that generative AI or AI-assisted technologies were not used in any way to prepare, write, or complete this manuscript. The authors confirm that they are the sole authors of this article and take full responsibility for the content therein, as outlined in COPE recommendations.

INFORMED CONSENT

The authors have obtained informed consent from all participants.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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