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# ON LOCAL VERTEX ANTIMAGIC TOTAL COLORING OF PATH, CYCLE, AND STAR GRAPHS WITH COMB OPERATION

**Taradita Ayitia Meisya Fendina[1], Desi Febriani Putri[2*], Wasono[3], Maria Alensia Deltin Dala[4]**

[1]Department of Mathematics, Mulawarman University, Indonesia
[2]Department of Mathematics, Mulawarman University, Indonesia
[3]Department of Mathematics, Mulawarman University, Indonesia
[4]Department of Mathematics, Mulawarman University, Indonesia
[*]Corresponding Author. E-mail: desifebrianip@fmipa.unmul.ac.id

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**ABSTRACT**

Misalkan $G(V,E)$ adalah suatu graf yang memiliki himpunan titik $V(G)$ dan himpunan sisi $E(G)$ dengan banyaknya titik dan sisi masing-masing dinyatakan oleh $|V(G)|$ dan $|E(G)|$. Fungsi bijektif $f: V(G) \cup E(G) \to \{1,2,3,\dots,(|V(G)| + |E(G)|)\}$ disebut pewarnaan lokal titik anti-ajaib total jika terdapat dua titik yang bertetangga $v_x$ dan $v_y$, dengan $w(v_x) \neq w(v_y)$, dan $w(v) = f(v) + \sum_{e \in E} f(e)$. Oleh karena itu, setiap pewarnaan lokal titik antiajaib total menginduksi pewarnaan titik dari $G$ dengan titik $v$ diberi warna $w(v)$. Penelitian ini penting karena memberikan kontribusi dalam pengembangan teori pewarnaan graf, khususnya pewarnaan lokal titik anti-ajaib total yang masih jarang diteliti. Pada penelitian ini membahas tentang pewarnaan lokal titik anti-ajaib total dari graf $P_m \rhd S_3$ dan $P_m \rhd C_3$ yang bertujuan untuk menentukan bilangan kromatiknya. Hasil dari penelitian ini adalah bilangan kromatik dari pewarnaan lokal titik antiajaib total pada graf $P_m \rhd S_3 = 4$ dan bilangan kromatik dari pewarnaan lokal titik antiajaib total pada graf $P_m \rhd C_3$, adalah 5 jika $m$ ganjil dan 7 jika $m$ genap.

Let $G(V,E)$ be a graph consisting of a set of vertices $V(G)$ and a set of edges $E(G)$ where the number of vertices and edges are denoted by $|V(G)|$ and $|E(G)|$, respectively. A bijective function $f: V(G) \cup E(G) \to \{1,2,3,\dots,(|V(G)| + |E(G)|)\}$ is defined as a local vertex antimagic total coloring if there exist two adjacent vertex $v_x$ and $v_y$ with $w(v_x) \neq w(v_y), w(v) = f(v) + \sum_{e \in E} f(e)$. Therefore, every local vertex antimagic total coloring produces a vertex coloring of the graph $G$, where each vertex $v$ is assigned a color corresponding to its weight $w(v)$. This research is essential as it contributes to development of graph coloring theory, particularly in the area of local vertex antimagic total coloring, which has been rarely studied. This research discusses the local vertex antimagic total coloring of $P_m \rhd S_3$ and $P_m \rhd C_3$ which aims to determine the chromatic number. The result of the research is the chromatic number of local vertex antimagic total coloring of $P_m \rhd S_3 = 4$ and the chromatic number of local vertex antimagic total coloring $P_m \rhd C_3$, is 5 if $m$ is odd and 7 if $m$ is even.

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## INTRODUCTION

Mathematics is the foundation of science. One branch of mathematics, discrete mathematics, which studies discrete objects, has developed rapidly in recent decades (Munir, 2010). Graph theory is a topic that attracts much interest because its models are beneficial to be applied in various aspects of everyday life (Nasir et al., 2022). Historically, graph theory was based on the problem of the Königsberg bridges in 1736 (Aziz, 2021). At that time, Euler thought of a way to cross all the bridges in the city of Kaliningrad, Russia, exactly once and return to the starting point (Ginting & Banjarnahor, 2010). Leonhard Euler was a mathematician who successfully found the solution to the problem by representing it as a graph (Fransiskus Fran, 2019). A graph is a set of edges and vertices $G(V, E)$, where $E$ is the set of edges and $V$ is the set of vertices (Daniel & Taneo, 2019). Theories developed in graph theory include labeling and coloring graphs (Majid et al., 2023).

Graph labeling is a bijective mapping that maps all graph (vertices, edges, and regions) into a set of positive integers and forms an arithmetic sequence (Magic Graphs, 2001). Based on the sum of weights, labeling can be classified into two categories: magic labeling and antimagic labeling. Magic labeling is the assignment of labels by arranging positive integers to objects so that those objects have the same weight sum. In contrasr, antimagic labeling is the assignment of labels by arranging positive integers so that those objects have different weight sums (Putri, 2016). A vertex antimagic total labeling on a graph with edges $e$ and vertices $v$ is a one-to-one mapping $f: V(G) \cup E(G) \longrightarrow \{1,2, \dots, |V(G) + E(G)|\}$ such that for each vertex $v \in V(G)$ is $w(v) = f(v) + \sum f(e)$ where the sum is taken over all edges adjacent to $v$ and all such weights are distinct (Prasetyo, 2012). Graph coloring is the assignment of colors to the elements of a graph such that every adjacent element has a different color (Maftukhah et al., 2020). Graph coloring is divided into three types: vertex coloring, edge coloring, and face coloring (Himayati et al., 2020).

Along with the advancement of science, the concepts of labeling and coloring have been combined into a new area of study in graph theory, namely local antimagic coloring. Previous research on local vertex antimagic coloring was published by Arumugam et al. in 2017. The concept of local vertex antimagic coloring involves labeling each edge of a graph and determining the weight at each vertex such that adjacent vertices receive different colors (Arumugam et al., 2017). Furthermore, research by Agustin et al. in 2017 examined local edge antimagic coloring, where labeling is applied to the graph in order to determine the weight of each edge such that adjacent edges have different colors (Agustin et al., 2017). This topic was further developed by Kurniawati in 2018, who studied local edge antimagic coloring using the comb operation on the graph $P_n \rhd H$ (Kurniawati et al., 2018). Subsequently, Putri et al. (2018) investigated the local antimagic total coloring of some families tree graphs. Research continued with Pawar and Singh (2023), who studied super local vertex antimagic total coloring in several families of graphs. In the same year, Gee-Choon Lau et al. Demonstrated that every graph admits a local antimagic total coloring and discussed some of its applications (Lau, et.al, 2023). The most recent research on super total local antimagic coloring of graphs was conducted by Pawar et al. (2025).

In this study, the focus is on determining the chromatic number of local antimagic total coloring using the comb operation. The choice of this operation is particularly interesting because no previous research has applied it to local vertex antimagic total coloring. Therefore, this study is expected to contribute to the understanding local vertex antimagic total coloring on graphs resulting from the comb operation and to open opportunities for further development of graph theory.

## METHOD

This research uses the pattern recognition and axiomatic-deductive method to solve the problem. The pattern detection approach is applied to determine and formulate the labeling structures of vertices and edges within the analyzed graph, thereby deriving a general formula for local vertex antimagic total labeling. Meanwhile, the axiomatic deductive method is a research approach that applies deductive reasoning in mathematical logic, relying on existing axioms or theorems to solve a problem. For further clarification regarding the research stages, see Figure 1.

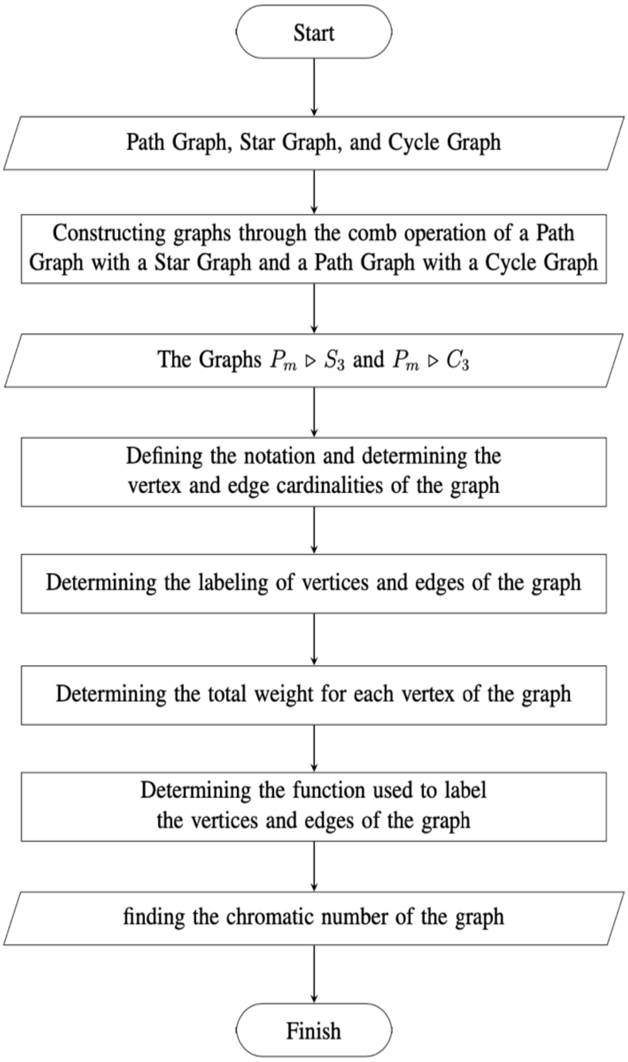


Figure 1. Research stages

## RESULTS

In this section, the results of the study on local vertex antimagic total coloring of the graphs $P_m \rhd S_3$ and $P_m \rhd C_3$ using the comb operation will be discussed.

### Local Vertex Antimagic Total Coloring on Graphs $P_m \rhd S_3$

The graph $P_m \rhd S_3$ is obtained from the comb product between the path graph $P_m$ and the star graph $S_3$ where $P_m$ consists of $m$ vertices and $S_3$ is a star graph composed of one central vertex and three pendant vertices. An example of the graph $P_m \rhd S_3$ can be seen in Figure 2.

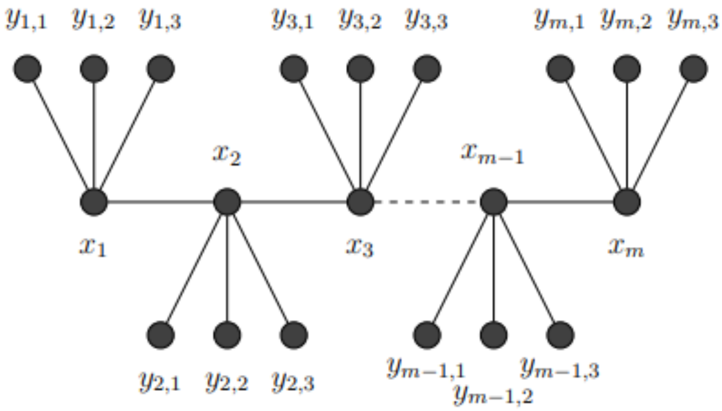


Figure 2. The graph of $P_m \rhd S_3$

**Theorem 1.** Let $P_m$ be a path graph and $S_3$ a star graph. For $m \geq 3$, the graph $P_m \rhd S_3$ has a chromatic number of local vertex antimagic total coloring $\chi_{lvat}(P_m \rhd S_3) = 4$.

**Proof.** Given the graph $P_m \rhd S_3$ with $m$ being a natural number, $m \geq 3$. The vertex set and the edge set of the graph $P_m \rhd S_3$ can be denoted respectively as follows:

$$V(P_m \rhd S_3) = \{x_i : 1 \leq i \leq m\} \cup \{y_{i,j} : 1 \leq i \leq m, 1 \leq j \leq 3\}$$

$$E(P_m \rhd S_3) = \{x_i x_{i+1} : 1 \leq i \leq m-1\} \cup \{x_i y_{i,j} : 1 \leq i \leq m, 1 \leq j \leq 3\}$$

Based on the vertex set and edge set above, the cardinalities of the vertices and edges of the graph $P_m \rhd S_3$ are respectively given by $|V(P_m \rhd S_3)| = 4m$ and $|E(P_m \rhd S_3)| = 4m - 1$.

Define a bijective labeling function on the graph $P_m \rhd S_3$, as follows:

$$g: \big(V(P_m \rhd S_3) \cup E(P_m \rhd S_3)\big) \rightarrow \{1,2,3, \dots, (8m-1)\}.$$

**Case 1** For $m$ odd

The first step begins with labeling all edges of the graph $P_m \rhd S_3$. The edge labeling function on $P_m \rhd S_3$ is defined as follows:

$$f(x_i x_{i+1}) = \begin{cases} m - \dfrac{i+1}{2}, & \text{for } 1 \leq i \leq m-2 \text{ and } i \text{ odd} \\ \dfrac{i}{2}, & \text{for } 1 \leq i \leq m-1 \text{ and } i \text{ even} \end{cases}$$

$$f(x_i y_{i,1}) = \begin{cases} \dfrac{7m+i-2}{2}, & \text{for } 1 \leq i \leq m \text{ and } i \text{ odd} \\ 3m + \dfrac{i-2}{2}, & \text{for } 1 \leq i \leq m \text{ and } i \text{ even} \end{cases}$$

$$f(x_i y_{i,2}) = \begin{cases} \dfrac{5m-i}{2}, & \text{for } 1 \leq i \leq m \text{ and } i \text{ odd} \\ 3m - \dfrac{i}{2}, & \text{for } 1 \leq i \leq m \text{ and } i \text{ even} \end{cases}$$

$$f(x_i y_{i,3}) = \begin{cases} (2m-1) - \dfrac{i-1}{2}, & \text{for } 1 \le i \le m \text{ and } i \text{ odd} \\ \dfrac{3m-i-1}{2}, & \text{for } 1 \le i \le m \text{ and } i \text{ even} \end{cases}$$

The second step is to assign labels to all vertices of the graph $P_m \rhd S_3$. The vertex labeling function is defined as follows:

$$f(v) = \begin{cases} 7m + \dfrac{i-1}{2}, & v = x_i, \text{for } 1 \le i \le m \text{ and } i \text{ odd} \\ \dfrac{15m+i-1}{2}, & v = x_i, \text{for } 1 \le i \le m-1 \text{ and } i \text{ even} \\ \dfrac{9m-i}{2}, & v = y_{i,1}, \text{for } 1 \le i \le m \text{ and } i \text{ odd} \\ \dfrac{11m+i-2}{2}, & v = y_{i,2}, \text{for } 1 \le i \le m \text{ and } i \text{ odd} \\ 6m + \dfrac{i-1}{2}, & v = y_{i,3}, \text{for } 1 \le i \le m \text{ and } i \text{ odd} \\ 5m - \dfrac{i}{2}, & v = y_{i,1}, \text{for } 1 \le i \le m-1 \text{ and } i \text{ even} \\ 5m + \dfrac{i-2}{2}, & v = y_{i,2}, \text{for } 1 \le i \le m-1 \text{ and } i \text{ even} \\ \dfrac{13m+i-1}{2}, & v = y_{i,3}, \text{for } 1 \le i \le m-1 \text{ and } i \text{ even} \end{cases}$$

Next, the vertex weights for the local vertex antimagic total coloring of the graph $P_m \rhd S_3$ are determined. The vertex weight function of $P_m \rhd S_3$ is defined as follows:

$$w(v) = \begin{cases} 16m-3, & v = x_i, \text{for } 1 \le i \le m \text{ and } i \text{ odd} \\ 16m-2, & v = x_i, \text{for } 1 \le i \le m-1 \text{ and } i \text{ even} \\ \dfrac{31m-5}{3}, & v = x_m \\ 8m-1, & v = y_{i,j}, \text{for } 1 \le i \le m-1, 1 \le j \le 3 \end{cases}$$

Based on the vertex weights $w(v)$ obtained above, it can be observed that the local vertex antimagic total coloring of the graph $P_m \rhd S_3$, for $m$ odd, has a chromatic number of 4, with weights $16m-3$, $16m-2$, $\frac{31m-5}{2}$, and $8m-1$. Therefore, the chromatic number of the local vertex antimagic total coloring of the graph $P_m \rhd S_3$ for $m$ odd, is $\chi_{lvat}(P_m \rhd S_3) = 4$.

An illustration of the local vertex antimagic total coloring of the graph $P_m \rhd S_3$ for $m$ odd $m = 5$ is shown in Figure 3.

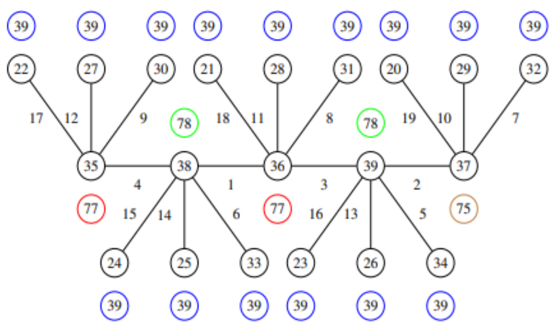


Figure 3. Illustrate the local vertex antimagic total coloring of $P_5 \rhd S_3$

Figure 3 presents an example of the local vertex antimagic total coloring of the graph $P_5 \rhd S_3$. As shown in Figure 3, the obtained chromatic numbers are $16m-3=77$, $16m-2=78$, $\frac{31m-5}{2}=75$, dan $8m-1=39$.

**Case 2** For $m$ even.

The first step begins with labeling all edges of the graph $P_m \rhd S_3$ as follows:

$$f(e)=\begin{cases} m-\dfrac{i+1}{2}; & e=x_ix_{i+1}, 1\le i\le m-1 \text{ dan } i \text{ odd}\\ \dfrac{i}{2}; & e=x_ix_{i+1}, 1\le i\le m-1 \text{ dan } i \text{ even}\\ \dfrac{7m+i-1}{2}; & e=x_iy_{i,1}, i \text{ odd}\\ \dfrac{5m-i-1}{2}; & e=x_iy_{i,2}, i \text{ odd}\\ (2m-1)-\dfrac{i-1}{2}; & e=x_iy_{i,3}, i \text{ odd}\\ 3m+\dfrac{i-2}{2}; & e=x_iy_{i,1}, i \text{ even}\\ 3m-\dfrac{i}{2}; & e=x_iy_{i,2}, i \text{ even}\\ \dfrac{3m-i}{2}; & e=x_iy_{i,3}, i \text{ even.} \end{cases}$$

The second step is to assign labels to the vertices of the graph $P_m \rhd S_3$ as follows:

$$f(v)=\begin{cases} \dfrac{15m+i-1}{2}; & v=x_i, 1\le i\le m-1 \text{ and } i \text{ odd}\\ 7m+\dfrac{i-2}{2}; & v=x_i, 1\le i\le m-1 \text{ and } i \text{ even}\\ \dfrac{9m-i-1}{2}; & v=y_{i,1}, i \text{ odd}\\ \dfrac{11m+i-1}{2}; & v=y_{i,2}, i \text{ odd}\\ 6m+\dfrac{i-1}{2}; & v=y_{i,3}, i \text{ odd}\\ 5m-\dfrac{i}{2}; & v=y_{i,1}, i \text{ even}\\ 5m+\dfrac{i-2}{2}; & v=y_{i,2}, i \text{ even}\\ \dfrac{13m+i-2}{2}; & v=y_{i,3}, i \text{ even.} \end{cases}$$

Next, the vertex weights for the local vertex antimagic total coloring of the graph $P_m \rhd S_3$ are determined. The vertex weight function of $P_m \rhd S_3$ is defined as follows:

$$w(v) = \begin{cases} \dfrac{33m-6}{2}, & v = x_i, \text{for } 1 \le i \le m \text{ and } i \text{ odd} \\ \dfrac{31m-4}{2}, & v = x_i, \text{for } 1 \le i \le m-1 \text{ and } i \text{ even} \\ 15m-2, & v = x_m \\ 8m-1, & v = y_{i,j}, \text{for } 1 \le i \le m-1, 1 \le j \le 3 \end{cases}$$

From the vertex weights $w(v)$ obtained above, it can be seen that the local vertex antimagic total coloring of the graph $P_m \rhd S_3$ for $m$ even has a chromatic number of 4, with weights $\frac{33m-6}{2}$, $\frac{31m}{2} - 2$, $15m - 2$, and $8m - 1$. Therefore, the chromatic number of the local vertex antimagic total coloring of the graph $P_m \rhd S_3$ for $m$ odd, is $\chi_{lvat}(P_m \rhd S_3) = 4$. ∎

Local vertex antimagic total coloring of the graph $P_m \rhd S_3$ for $m$ even, $m = 6$ can be illustrated and shown in Figure 4.

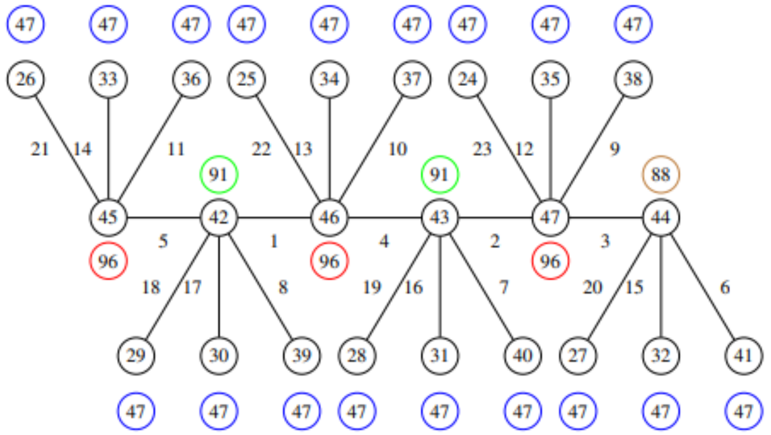

Figure 4. Local vertex antimagic total coloring of the graph $P_6 \rhd S_3$

Figure 4 illustrates an example of the local vertex antimagic total coloring on the graph $P_6 \rhd S_3$. Based on Figure 4, the obtained chromatic numbers are $\frac{33m-6}{2} = 96$, $\frac{31m}{2} - 2 = 91$, $15m - 2 = 88$, and $8m - 1 = 47$.

### Local Vertex Antimagic Total Coloring on Graphs $P_m \rhd C_3$

The graph $P_m \rhd C_3$ is obtained from the comb operation between the path graph $P_m$ and the cycle graph $C_3$ where $P_m$ consists of $m$ vertices and $C_3$ is a cycle graph with three vertices. An illustration of the graph $P_m \rhd C_3$ is presented in Figure 5.

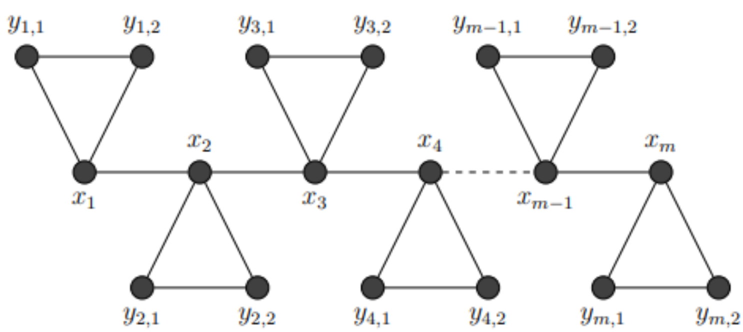


Figure 5. The Graph of $P_m \rhd C_3$

**Theorem 2** Let $P_m$ be a path graph and $C_3$ a cycle graph. For $m \geq 3$, the graph $P_m \rhd C$ has the following total local antimagic vertex coloring chromatic number:

$$\chi_{lvat}(P_m \rhd C_3) = \begin{cases} 5; & \text{for } m \text{ odd} \\ 7; & \text{for } m \text{ even} \end{cases}$$

**Proof.** Given the graph $P_m \rhd C_3$ with $m$ being a natural number, $m \geq 3$. The vertex set and the edge set of the graph $P_m \rhd C$ can be denoted respectively as follows:

$$V(P_m \rhd C_3) = \{x_i: 1 \leq i \leq m\} \cup \{y_{i,j}: 1 \leq i \leq m, 1 \leq j \leq 2\}$$

$$E(P_m \rhd C_3) = \{x_i x_{i+1}: 1 \leq i \leq m-1\} \cup \{x_i y_{i,j}: 1 \leq i \leq m, 1 \leq j \leq 2\} \cup \{y_i y_{i,j+1}: 1 \leq i \leq m, j = 1\}.$$

Based on the vertex set and edge set above, the cardinalities of the vertices and edges of the graph $P_m \rhd C_3$ are respectively given by $|V(P_m \rhd C_3)| = 4m$ and $|E(P_m \rhd C_3)| = 4m - 1$.

Define a bijective labeling function on the graph $P_m \rhd S_3$, as follows:

$$g: \left(V(P_m \rhd C_3) \cup E(P_m \rhd C_3)\right) \to \{1,2, \dots, (7m-1)\}.$$

**Case 1** For $m$ odd

The first step begins with labeling all edges of the graph $P_m \rhd C_3$. The edge labeling function on $P_m \rhd C_3$ is defined as follows:

$$f(e) = \begin{cases} m - \dfrac{i+1}{2}; & e = x_i x_{i+1}, 1 \leq i \leq m-1 \text{ and } i \text{ odd} \\ \dfrac{i}{2}; & e = x_i x_{i+1}, 1 \leq i \leq m-1 \text{ and } i \text{ even} \\ \dfrac{3m+i-2}{2}; & e = y_{i,1} y_{i,2}, i \text{ odd} \\ 3m + \dfrac{i-1}{2}; & e = x_i y_{i,1}, i \text{ odd} \\ 2m + \dfrac{i-1}{2}; & e = x_i y_{i,2}, i \text{ odd} \\ m + \dfrac{i-2}{2}; & e = y_{i,1} y_{i,2}, i \text{ even} \\ \dfrac{7m+i-1}{2}; & e = x_i y_{i,1}, i \text{ even} \\ \dfrac{5m+i-1}{2}; & e = x_i y_{i,2}, i \text{ even.} \end{cases}$$

The second step is to assign labels to all vertices of the graph $P_m \rhd C_3$. The vertex labeling function is defined as follows:

$$f(v) = \begin{cases} 7m - i, & v = x_i, 1 \leq i \leq m \\ 6m - i, & v = y_{i,1}, 1 \leq i \leq m \\ 5m - i, & v = y_{i,2}, 1 \leq i \leq m \end{cases}$$

Next, the vertex weights for the local vertex antimagic total colouring of the graph $P_m \rhd C_3$ are determined. The vertex weight function of $P_m \rhd C_3$ is defined as follows:

$$w(v)=\begin{cases}13m-2, & v=x_i, i \text{ odd}\\ 14m-1, & v=x_i, i \text{ even}\\ \dfrac{25m-3}{2}, & v=x_m\\ \dfrac{21m-3}{2}, & v=y_{i,1}, 1\le i\le m\\ \dfrac{17m-3}{2}, & v=y_{i,2}, 1\le i\le m\end{cases}$$

From the vertex weights $w(v)$ obtained above, it can be seen that the local vertex antimagic total coloring of the graph $P_m \rhd C_3$, for $m$ odd, has a chromatic number of 5, with weights $13m-2$, $14m-1$, $\frac{25m-3}{2}$, $\frac{21m-3}{2}$, and $\frac{17m-3}{2}$.

An illustration of the local vertex antimagic total coloring of the graph $P_m \rhd C_3$ for $m$ odd $m=5$ is shown in Figure 6.

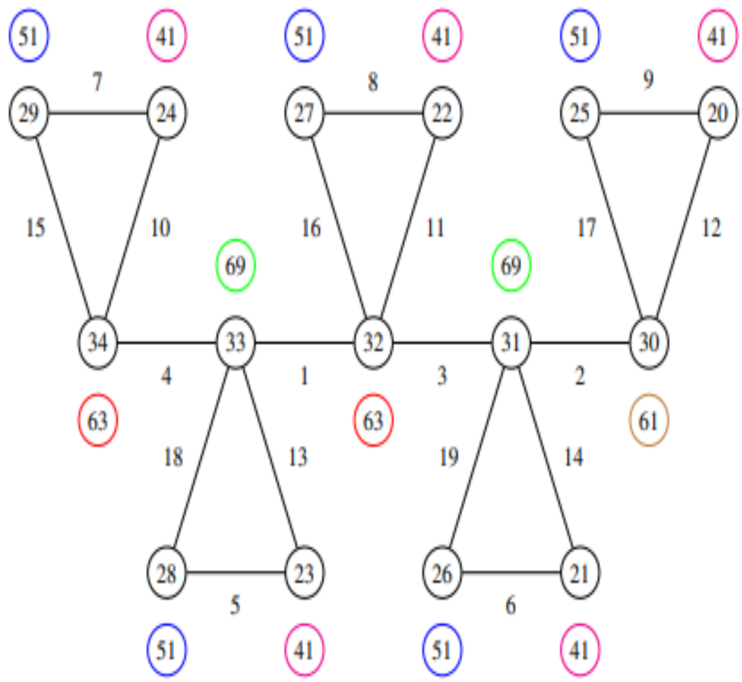

Figure 6. Local vertex antimagic total coloring of the graph $P_5 \rhd C_3$

Figure 6 presents an example of the local vertex antimagic total coloring of the graph $P_5 \rhd C_3$. As shown in Figure 5, the obtained chromatic numbers are $13m-2=63$, $14m-1=69$, $\frac{25m-3}{2}=61$, $\frac{21m-3}{2}=51$, and $\frac{17m-3}{2}=41$. Therefore, the chromatic number of the total local antimagic vertex coloring of the graph $P_m \rhd C_3$ is $\chi_{lvat}(P_5 \rhd C_3)=5$.

**Case 2** For $m$ even

The first step begins with labeling all edges of the graph $P_m \rhd C_3$. The edge labeling function on $P_m \rhd C_3$ is defined as follows:

$$f(e)=\begin{cases} m-\dfrac{i+1}{2}; & e=x_ix_{i+1}, 1\le i\le m-1 \text{ and } i \text{ odd}\\ \dfrac{i}{2}; & e=x_ix_{i+1}, 1\le i\le m-1 \text{ and } i \text{ even}\\ \dfrac{3m+i-1}{2}; & e=y_{i,1}y_{i,2}, i \text{ odd}\\ 3m+\dfrac{i-1}{2}; & e=x_iy_{i,1}, i \text{ odd}\\ 2m+\dfrac{i-1}{2}; & e=x_iy_{i,2}, i \text{ odd}\\ m+\dfrac{i-2}{2}; & e=y_{i,1}y_{i,2}, i \text{ even}\\ \dfrac{7m+i-2}{2}; & e=x_iy_{i,1}, i \text{ even}\\ \dfrac{5m+i-2}{2}; & e=x_iy_{i,2}, i \text{ even.} \end{cases}$$

The second step is to assign labels to all vertices of the graph $P_m \rhd C_3$. The vertex labeling function is defined as follows:

$$f(v)=\begin{cases} 7m-i, & v=x_i, 1\le i\le m\\ 6m-i, & v=y_{i,1}, 1\le i\le m\\ 5m-i, & v=y_{i,2}, 1\le i\le m \end{cases}$$

Next, the vertex weights for the local vertex antimagic total colouring of the graph $P_m \rhd C_3$ are determined. The vertex weight function of $P_m \rhd C_3$ is defined as follows:

$$w(v)=\begin{cases} 13m-2; & v=x_i, i \text{ odd}\\ 14m-1; & v=x_i, i \text{ even}\\ \dfrac{27m}{2}-2; & v=x_m\\ \dfrac{21m-2}{2}; & v=y_{i,1}, i\ odd\\ \dfrac{17m-2}{2}; & v=y_{i,2}, i \text{ odd}\\ \dfrac{21m-4}{2}; & v=y_{i,1}, i \text{ even}\\ \dfrac{17m-4}{2}; & v=y_{i,2}, i \text{ even} \end{cases}$$

From the vertex weights $w(v)$ obtained above, it can be seen that the local vertex antimagic total coloring of the graph $P_m \rhd C_3$, for $m$ even, has a chromatic number of 7, with weights $13m-2$, $14m-1$, $\frac{27m}{2}-2$, $\frac{21m-2}{2}$, $\frac{17m-2}{2}$, $\frac{21m-4}{2}$, and $\frac{17m-4}{2}$. Therefore, the chromatic number of the total local antimagic vertex coloring of the graph $P_m \rhd C_3$ is $\chi_{lvat}(P_6 \rhd C_3)=7$.

An illustration of the local vertex antimagic total coloring of the graph $P_m \rhd C_3$ for $m$ even $m=7$ is shown in Figure 7.

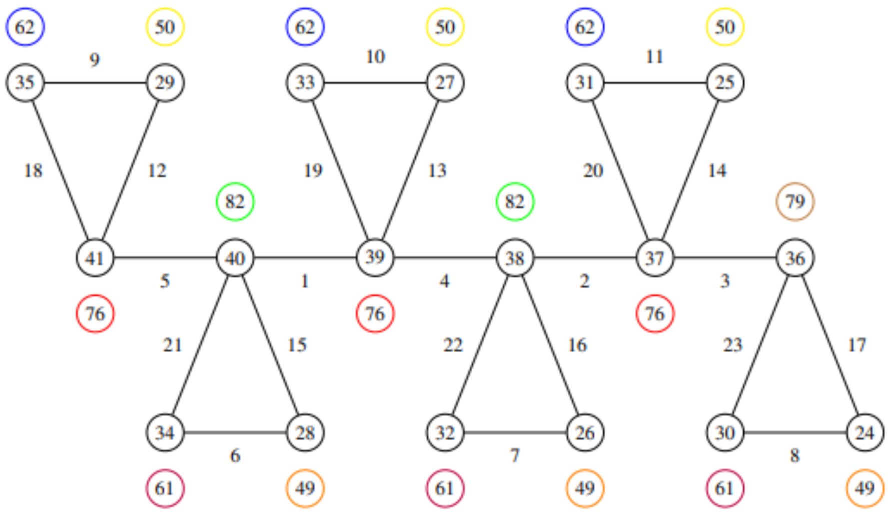


Figure 7. Local vertex antimagic total coloring of the graph $P_6 \rhd C_3$

Figure 7 presents an example of the local vertex antimagic total coloring of the graph $P_6 \rhd C_3$. As shown in Figure 6, the obtained chromatic numbers are $13m - 2 = 76$, $14m - 2 = 82$, $\frac{27m}{2} - 2 = 79$, $\frac{21m-2}{2} = 62$, $\frac{17m-2}{2} = 50$, $\frac{21m-4}{2} = 61$, and $\frac{17m-4}{2} = 49$.

## CONCLUSION

Based on the research results on local vertex antimagic total coloring for several graphs obtained from the comb operation, it can be concluded that the chromatic number of the total local antimagic vertex coloring for the graph $P_m \rhd S_3$ is 4 and for the $P_m \rhd C_3$ is 5 when m is odd and 7 when m is even.

**Open Problem:** Determine chromatic number of local vertex antimagic total coloring for graphs $P_m \rhd S_n$ and $P_m \rhd C_n$.

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## INFORMED CONSENT

The authors have obtained informed consent from all participants.

## CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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