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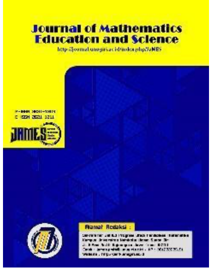

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NUMERICAL MODELING OF OIL PALM PLANTATION AREA GROWTH IN INDONESIA USING VERHULST MODEL

Ajril Abyad Al Haq[1*] , Muna Afdi Muniroh[1]

[1]Department of Mathematics, Universitas PGRI Ronggolawe, Indonesia

[*]Corresponding Author. E-mail: abyadalhaq02@gmail.com

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ABSTRACT

The expansion of oil palm plantation area in Indonesia exhibits nonlinear growth behavior influenced by environmental, spatial, and policy constraints. This study aims to model the growth of oil palm plantation area using the Verhulst (logistic) growth model and to obtain numerical solutions of the resulting nonlinear differential equation. A quantitative approach is employed, in which the fourth-order Runge–Kutta (RK4) method is used to generate the required initial values, followed by two predictor–corrector schemes, namely the Adams–Bashforth–Backward Differentiation Formula (AB4–BDF4) and the Milne–Backward Differentiation Formula (Milne–BDF4), which are compared to approximate the long-term growth dynamics. The analysis utilizes annual oil palm plantation area data from 2017 to 2024 obtained from Statistics Indonesia (BPS). Numerical results indicate that all methods produce stable solutions, with relative errors remaining bounded and non-divergent throughout the simulation, on the order of 10^{-6}. On average, the AB4–BDF4 scheme yields a relative error of 3.2477330×10^{-6}, compared to 5.9206896×10^{-6} for the Milne–BDF4 scheme, corresponding to an accuracy improvement of approximately 1.8 times. The estimated intrinsic growth rate is $r \approx 0.15$, and the estimated carrying capacity is 20 million hectares, representing the maximum sustainable oil palm plantation area under current growth trends.

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INTRODUCTION

Oil palm plantations are one of the most important components of Indonesia's agricultural sector and play a strategic role in supporting national economic growth and the global palm oil supply chain. The expansion of oil palm plantations in Indonesia has increased rapidly over the past 20 years due to growing demand from the food, oleochemical, and bioenergy sectors (Isra et al., 2025). Economic development has benefited greatly from this expansion, but uncontrolled land growth has also caused environmental degradation, loss of biodiversity, land conversion, and deforestation (Mahtta et al., 2022).

The expansion of oil palm plantations cannot continue indefinitely. Land growth is naturally and institutionally limited by a number of factors, including limited land availability, environmental carrying capacity, climate variability, land use conflicts, and government regulations, including those related to spatial planning and forest protection (Shahputra & Zen, 2018). Excessive clearing of new land can exacerbate ecological damage, increase greenhouse gas emissions, and threaten the long-term viability of agriculture if land expansion is not controlled. To plan for sustainable land use and create environmentally friendly regulations, it is essential to understand the long-term growth patterns of oil palm plantations (Hidayat & Fajri, 2026).

From a mathematical perspective, oil palm plantation land growth exhibits nonlinear characteristics with an upper limit, where the growth rate gradually decreases as the land area approaches the maximum available limit (Maulidya et al., 2021). This pattern is consistent with Verhulst's logistic growth concept, which uses a carrying capacity parameter to describe spatial, regulatory, and environmental constraints. By considering the impact of saturation and calculating the maximum area of oil palm plantations that can be maintained under current ecological and policy conditions, the Verhulst model offers a realistic framework for explaining land expansion dynamics (Susanti & Maryudi, 2016).

Previous research on oil palm expansion has primarily focused on production performance, productivity improvement, and economic impacts using statistical, econometric, or time-series approaches (Okarda et al., 2024). Although Abanum (2020) formulated oil palm dynamics using a nonlinear differential equation model, the model did not explicitly incorporate the saturation effect associated with limited land availability. This study adopts the Verhulst growth model, which introduces a carrying capacity to represent the finite availability of plantation land, thereby providing a more realistic description of long-term plantation expansion.

Numerical methods such as the fourth-order Runge-Kutta (RK4), Adams-Bashforth (AB4), Backward Differentiation Formula (BDF4), and Milne predictor-corrector are widely used to solve nonlinear ordinary differential equations (Ibrahim et al., 2024; Olatunji & Akeju, 2025). However, comparative applications of higher-order predictor-corrector methods to the Verhulst model for palm plantation growth remain limited. In this study, the fourth-order Runge-Kutta (RK4) method is used to generate the initial values required by the predictor-corrector schemes. Subsequently, the AB4-BDF4 and Milne-BDF4 methods are applied to obtain the numerical solutions of the Verhulst model. The predictor-corrector framework combines explicit prediction with implicit correction, offering improved stability over single-step methods when applied to nonlinear, saturating growth models such as the Verhulst equation, and providing an efficient approach for long-term numerical simulations of nonlinear growth models. The numerical accuracy and computational performance of the AB4-BDF4 and Milne-BDF4 methods are then compared.

Accordingly, this study aims to model the expansion of oil palm plantation area in Indonesia using the Verhulst growth model and to obtain numerical solutions of the resulting nonlinear differential equation using the RK4, Milne predictor–corrector, AB4, and BDF4 methods. In addition, this study seeks to estimate the carrying capacity representing the maximum sustainable plantation area under environmental, spatial, and policy constraints, as well as to compare the numerical performance of the applied methods.

METHOD

This study examines the expansion of Indonesia's oil palm plantation area using a quantitative approach based on mathematical modeling and numerical methods. Nonlinear growth with a limiting capacity is represented by the Verhulst logistic growth model, and numerical solutions are achieved by first applying the fourth-order Runge–Kutta (RK4) method to generate the required initial values, followed by two predictor–corrector schemes, namely the Adams–

Bashforth–Backward Differentiation Formula (AB4–BDF4) and the Milne–Backward Differentiation Formula (Milne–BDF4). The following is a summary of the research process, illustrated in Figure 1.

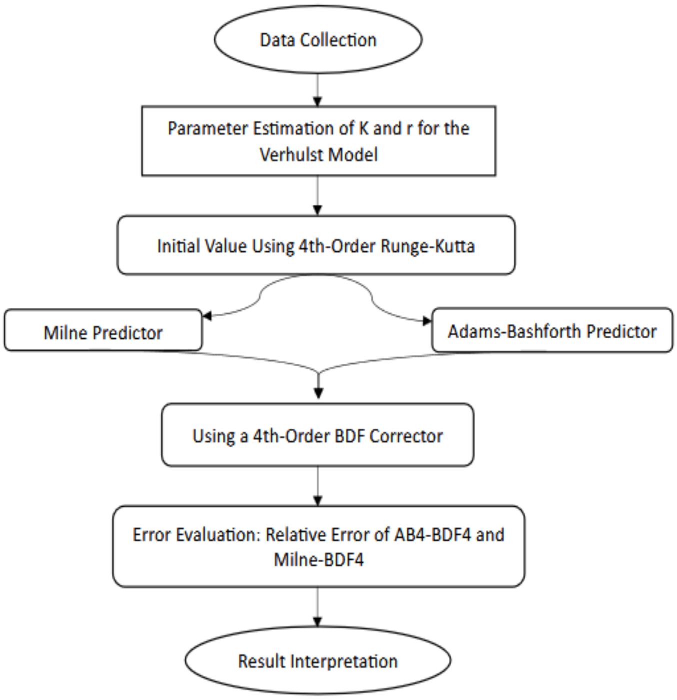

Figure 1. Research workflow

Figure 1 outlines the sequential stages of the research, from data acquisition through to result interpretation. Each stage is described in detail below.

1. Data Collection
   The study's data set includes Indonesia's annual oil palm plantation area, which was sourced from Statistics Indonesia's official website (BPS). The time frame covered by the dataset is 2017–2024 (BPS Indonesia, 2025). The 2017–2024 period was selected because it captures a sustained, uninterrupted expansion phase without irregular fluctuations, making it representative for modeling the underlying growth dynamics using the Verhulst framework.
2. Verhulst Model
   The Verhulst (logistic) growth model, introduced by Pierre Verhulst, incorporates environmental carrying capacity and is used in this study to describe the growth of Indonesia's oil palm plantation area, where expansion slows as the area approaches its maximum limit. (Anggreini et al., 2019). The Verhulst growth model is expressed as follows:
   $$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \tag{1}$$
   $P(t)$ represents the oil palm plantation area at time t, r is the intrinsic growth rate, and K is the carrying capacity (maximum sustainable area). The growth rate r is estimated using Equation (2), based on the plantation area of the first two years in the dataset (2017 and 2018), giving $r = (1/1)\ln(14{,}326{,}350/12{,}383{,}101) \approx 0.15$. The first-year interval was chosen to represent the initial growth rate prior to the onset of saturation effects, since later intervals already reflect the influence of the approaching carrying capacity and would understate the intrinsic growth rate r.
   $$P = P_0 e^{rt} \tag{2}$$

$$r = \frac{1}{t}\ln\left(\frac{P}{P_0}\right)$$

3. Fourth-order Runge-Kutta Method
   This method was discovered by Carl Runge (1856-1927) and Wilhelm Kutta (1867-1944), who were German mathematicians. The Runge-Kutta method is another alternative to the Taylor Series method used to solve ordinary differential equations (Suryani et al., 2023). The form of the Fourth Order Runge-Kutta method equation is as follows:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h \tag{3}$$

   in this case k is:

$$k_1 = f(x_i, y_i)$$
$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$$
$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$$
$$k_4 = f(x_i + h, y_i + k_3 h)$$

4. Adam-Bashforth
   The Adams–Bashforth method is an explicit multistep numerical technique originally developed by John Couch Adams and Francis Bashforth in the nineteenth century for solving ordinary differential equations. To get fourth-order accuracy, the fourth-order Adams–Bashforth approach (AB4) makes use of data from four prior solution points (Raming et al., 2024). However, the Adams–Bashforth fourth-order technique is only used as a predictor in this study:

$$y(x_{n+1}) = y(x_n) + \int_{x_n}^{x_{n+1}} f(x, y(x))dx \tag{4}$$

   Fourth-Order Adam-Bashforth Predictor Equation:

$$y_{n+1}^P = y_n + \frac{h}{24}(-9f_{n-3} + 37f_{n-2} - 59f_{n-1} + 55f_n) \tag{5}$$

5. Milne Predictor Method
   The Milne method is applied using only the predictor formula to estimate the next solution value of the Verhulst model. The Milne predictor is an explicit multistep method obtained from the integration of $f(x, y(x))$ over the interval $[x_{n-3}, x_{n+1}]$, with the required initial values generated using the RK4 method (Adebisi et al., 2025).

$$y(x_{n+1}) = y(x_{n-3}) + \int_{x_{n-3}}^{x_{n+1}} f(x, y(x))dx \tag{6}$$

   Milne's Predictor Equation:

$$y_{n+1}^P = y_{n-3} + \frac{4h}{3}(2f_{n-2} - f_{n-1} + 2f_n) \tag{7}$$

6. Backward Differentiation Formula (BDF4)
   The Fourth-Order Backward Differentiation Formula (BDF4) is used as a correction method to improve the accuracy and stability of predicted solutions (Anie, 2016). BDF4 is an implicit multistep method that uses information from previous solution points to improve predicted values (Audu et al., 2022). The BDF4 scheme is expressed as:

$$\frac{25}{12}y_{n+4} - 4y_{n+3} + 3y_{n+2} - \frac{4}{3}y_{n+1} + \frac{1}{4}y_n = hf(t_{n+4}, y_{n+4}) \tag{8}$$

where h denotes the time step size.

7. Relative Error Calculation
   The final step involves calculating the relative error between the predictor and corrector values at each time step to evaluate numerical accuracy (Tukan et al., 2024). The relative error is calculated as $Ep = |p - \hat{p}|/|p|$, where p is the corrector value and $\hat{p}$ is the predictor value, with $p \neq 0$.

RESULTS

This data analysis focuses on modeling and numerically solving the growth dynamics of oil palm plantation area in Indonesia using a mathematical growth framework. In order to reflect recent patterns in land expansion over time, the analysis uses annual data on the total area of oil palm plantations for the years 2017 through 2024. The long-term trend of plantation area growth under current environmental, policy, and economic restrictions is captured by these data. Parameter estimate, numerical computation, and comparative analysis of the used numerical methods are all based on the dataset. Table 1 displays the comprehensive data used in the numerical computations:

Table 1. Total Area of Oil Palm Plantations in Indonesia

No.	Year	Total palm oil plantation area (ha)
1.	2017	12,383,101
2.	2018	14,326,350
3.	2019	14,456,612
4.	2020	14,586,597
5.	2021	14,621,693
6.	2022	15,338,556
7.	2023	15,928,712
8.	2024	16,013,039

Table 1 shows the development of oil palm plantation area in Indonesia during the period 2017–2024. Although the growth rate tended to slow down in some years due to various land and policy constraints, the area of oil palm plantations rose rather steadily during this time, from 12,383,101 hectares in 2017 to 16,013,039 hectares in 2024. The most notable increase occurred between 2017 and 2018, when the area expanded by 1,943,249 hectares, after which growth slowed markedly, averaging under 1% annually through 2021 before picking up again in 2022–2023.

Initial Values of the Vehulst Model Using the Fourth-Order Runge-Kutta Method

The initial value is found using the Fourth Order Runge-Kutta Method with the following equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \tag{9}$$

Next, use equation 2 to determine the value of r (land area growth rate):

$$P = P_0 e^{rt} \tag{10}$$

$$r = \frac{1}{t}\ln\left(\frac{P}{P_0}\right)$$

$$r = \frac{1}{1}\ln\left(\frac{14{,}326{,}350}{12{,}383{,}101}\right) \approx 0.15$$

Through trial and error, the maximum land capacity, or K, is determined by entering the estimated value of K into the Verhulst model (Robbaniyyah et al., 2025). With only a handful of yearly observations, fitting K through a formal optimization procedure such as least squares tends to produce unstable results. A trial-and-error approach, by contrast, allows the value to be adjusted gradually until the model curve visibly aligns with the observed saturation pattern — a pragmatic choice given the size of the dataset. From 2017 to 2024, the area of oil palm plantations in Indonesia will remain below $20{,}000{,}000$ hectares. Suppose $K = 20{,}000{,}000$. When the values of r and K are entered into equation 9, the result is:

$$\frac{dP}{dt} = 0.15P\left(1 - \frac{P}{20{,}000{,}000}\right) \tag{11}$$

The following are the initial values of P_1, P_2, and P_3 obtained from the Fourth Order Runge-Kutta Method on the interval [1,16] with step size $h = 1$, and $P_0 = 12{,}383{,}101$. Table 2 shows the initial values of the RK4 method:

Table 2. Initial Value Using the Fourth-Order Runge-Kutta Method

n	***t_n***	***P_n***	***f(t,P)***
0	1.	12,383,101	687,443.2185886
1	2.	13,057,634.1096861	660,695.2949763
2	3.	13,702,809.2846490	628,905.9156114
3	4.	14,314,134.6654177	593,185.9942268

Table 2 presents the initial time points, estimated oil palm plantation areas, and corresponding growth rates obtained using the fourth-order Runge–Kutta method.

Fourth-Order Adams–Bashforth Predictor and BDF4 Corrector Solution of the Verhulst Growth Model for Oil Palm Plantation Area in Indonesia

The numerical solution, which provides an estimate of the oil palm plantation area in Indonesia, is obtained using the fourth-order Adams–Bashforth-BDF4 method after the initial values are generated by the fourth-order Runge–Kutta method and substituted into the Adams–Bashforth-BDF4 equations. The result for $n = 3$ with $P_3 = 14{,}314{,}134.67$ is as follows:

$$P_n^{(0)} = P_n + \frac{h}{24}(-9f_{n-3} + 37f_{n-2} - 59f_{n-1} + 55f_n) \tag{12}$$

After obtaining the initial value from Runge-Kutta, substitute it into equation (12):

$$\begin{gathered} P_4^0 = P_3 + \frac{h}{24}(-9f_0 + 37f_1 - 59f_2 + 55f_3) \\ P_4^0 = 14{,}888{,}239.5657603 \end{gathered} \tag{13}$$

Next, find the value of f_4^0, obtained:

$$f_4^0 = f(t_4, P_4^0) = 554{,}679.7472410 \tag{14}$$

The Adam-Bashforth predictor value was then corrected using the BDF4 corrector equation as follows:

$$\frac{25}{12}P_4 - 4P_3 + 3P_2 - \frac{4}{3}P_1 + \frac{1}{4}P_0 = hf_4 \tag{15}$$

Multiply both sides of equation (15) by $\frac{12}{25}$, then the equation becomes:

$$P_4 = \frac{12}{25}\left[4P_3 - 3P_2 + \frac{4}{3}P_1 - \frac{1}{4}P_0 + hf_4\right] \tag{16}$$

After obtaining Equation (16), substitute the results from f4 and the initial value of Rk4 to obtain the corrector results:

$$P_4^{BDF4} = 14{,}888{,}253.1765822$$

Then, the AB4 predictor and BDF4 corrector values will be used to find the relative error, obtained as follows:

$$\frac{|P_4^{BDF4} - P_4^0|}{|P_4^{BDF4}|} = \frac{|14{,}888{,}253.18 - 14{,}888{,}239.57|}{|14{,}888{,}253.18|} = 9.1389220 \times 10^{-7} \quad (17)$$

The numerical results obtained using the fourth-order Adams–Bashforth predictor and the fourth-order Backward Differentiation Formula corrector (AB4–BDF4) are presented in Table 3, including the predicted and corrected values at each time step along with their relative errors.

Table 3. Numerical Solution and Relative Error Using AB4 and BDF4 Method

n	t_n	P_4^0	P_n^{BDF4}	Relative Error
0	1.		12,383,101	
1	2.		13,057,634.1096861	
2	3.		13,702,809.2846490	
3	4.		14,314,134.6654177	
4	5.	14,888,239.5657603	14,888,253.1720187	9.1389220×10^{-7}
5	6.	15,422,947.8861361	15,422,932.9333459	9.6951665×10^{-7}
6	7.	15,917,057.9076184	15,917,013.5988660	2.7837353×10^{-6}
7	8.	16,370,369.0919226	16,370,301.3720848	4.1367496×10^{-6}
8	9.	16,783,509.2072525	16,783,427.6853721	4.8572843×10^{-6}
9	10.	17,157,778.4738731	17,157,692.3351440	5.0204146×10^{-6}
10	11.	17,494,991.0654382	17,494,907.1738303	4.7952017×10^{-6}
11	12.	17,797,327.1908041	17,797,250.0736437	4.3330942×10^{-6}
12	13.	18,067,202.2721833	18,067,134.6721900	3.7416001×10^{-6}
13	14.	18,307,155.7103232	18,307,099.0074411	3.0973166×10^{-6}
14	15.	18,519,759.8689410	18,519,714.3449547	2.4581365×10^{-6}
15	16.	18,707,548.7730298	18,707,513.8675356	1.8658542×10^{-6}

The predictor–corrector findings utilizing the BDF4 corrector and the fourth-order Adams-Bashforth predictor are shown in Table 3. In addition to the projected values, corrected solutions, and associated relative errors, values for $n = 4$ to $n = 15$ are displayed. It is evident that Indonesia's oil palm farms have been growing in size over time. Table 4 is a comparison of the estimated solutions of the Verhulst Model using the AB4-BDF4 method, predictor-corrector with actual data starting from $n = 4$ to $n = 7$.

Table 4. Estimated Area of Oil Palm Plantations

n	t_n	P_4^0	P_n^{BDF4}	Plantation Area Data (ha)
0	2017		12,383,101	12,383,101
1	2018		13,057,634.1096861	14,326,350
2	2019		13,702,809.2846490	14,456,612
3	2020		14,314,134.6654177	14,586,597
4	2021	14,888,239.5657603	14,888,253.1720187	14,621,693
5	2022	15,422,947.8861361	15,422,932.9333459	15,338,556
6	2023	15,917,057.9076184	15,917,013.5988660	15,928,712
7	2024	16,370,369.0919226	16,370,301.3720848	16,013,039
8	2025	16,783,509.2072525	16,783,427.6853721	
9	2026	17,157,778.4738731	17,157,692.3351440	
10	2027	17,494,991.0654382	17,494,907.1738303	
11	2028	17,797,327.1908041	17,797,250.0736437	
12	2029	18,067,202.2721833	18,067,134.6721900	
13	2030	18,307,155.7103232	18,307,099.0074411	
14	2031	18,519,759.8689410	18,519,714.3449547	
15	2032	18,707,548.7730298	18,707,513.8675356	

Milne Fourth Order Predictor and BDF4 Corrector Solution of the Verhulst Growth Model for Oil Palm Plantation Areas in Indonesia

The numerical solution used to estimate the area of oil palm plantations is obtained by applying the Adams–Bashforth–BDF4 method after determining the initial values. The values of P_1, P_2, and P_3, along with the value of $f_n(t_n, P_n)$, are first calculated using the fourth-order Runge–Kutta method and then substituted into the Milne-BDF4 equations. For $n = 3$ with $P_0 = 12{,}383{,}101$ is presented as follows:

$$P_{n+1}^{(0)} = P_{n-3} + \frac{4h}{3}(2f_{n-2} - f_{n-1} + 2f_n) \tag{18}$$

$$P_4^{(0)} = P_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3) \tag{19}$$

$$P_4^{(0)} = 14{,}888{,}243.2170596$$

After obtaining $P_4^{(0)}$, it is used to find the value of f_4^0, obtained:

$$f_4^0 = f\left(t_4, P_4^{(0)}\right) = 554{,}679.7512729 \tag{20}$$

The Milne predictor is then refined using the BDF4 corrector to obtain:

$$P_4 = \frac{12}{25}\left[4P_3 - 3P_2 + \frac{4}{3}P_1 - \frac{1}{4}P_0 + hf_4\right] \tag{21}$$

$$P_4^{BDF4} = 14{,}888{,}253.1785175$$

Then, after obtaining the values from the Milne predictor and BDF4 corrector, use them to calculate the relative error, resulting in:

$$\frac{|P_4^{BDF4} - P_4^0|}{|P_4^{BDF4}|} = \frac{|14{,}888{,}253.18 - 14{,}888{,}243.22|}{|14{,}888{,}253.18|} = 6.6864520 \times 10^{-7}$$

The numerical results obtained using the Milne predictor and the fourth-order Backward Differentiation Formula corrector (Milne–BDF4) are presented in Table 5, including the predicted and corrected values at each time step along with their relative errors.

Table 5. Numerical Solution and Relative Error Using Milne and BDF4 Method

n	t_n	P_4^0	P_n^{BDF4}	**Relative Error**
0	1.		12,383,101	
1	2.		13,057,634.1096861	
2	3.		13,702,809.2846490	
3	4.		14,314,134.6654177	
4	5.	14,888,243.2170596	14,888,253.1720187	6.6864520×10^{-7}
5	6.	15,422,947.8596378	15,422,932.9333459	9.6779854×10^{-7}
6	7.	15,917,059.5759334	15,917,013.5988660	2.8885486×10^{-6}
7	8.	16,370,382.8922089	16,370,301.3720848	4.9797571×10^{-6}
8	9.	16,783,546.4880020	16,783,427.6853721	7.0785677×10^{-6}
9	10.	17,157,840.6636662	17,157,692.3351440	8.6450158×10^{-6}
10	11.	17,495,070.5515236	17,494,907.1738303	9.3385859×10^{-6}
11	12.	17,797,413.8486259	17,797,250.0736437	9.2022634×10^{-6}
12	13.	18,067,288.0006937	18,067,134.6721900	8.4865977×10^{-6}
13	14.	18,307,235.3664067	18,307,099.0074411	7.4484202×10^{-6}
14	15.	18,519,830.4845348	18,519,714.3449547	6.2711324×10^{-6}
15	16.	18,707,608.7696813	18,707,513.8675356	5.0729427×10^{-6}

The numerical results obtained from the Milne–BDF4 predictor–corrector scheme are summarized in Table 3. This table presents the predicted values, corrected solutions, and relative errors for $n = 4$ to $n = 15$. The results demonstrate an increasing trend in the oil palm plantation area in Indonesia over the observed period. Furthermore, Table 6 compares the numerical solutions of the Verhulst model generated using the Milne–BDF4 method with the corresponding actual data for $n = 4$ to $n = 7$.

Table 6. Estimated Area of Oil Palm Plantations

n	t_n	P_4^0	P_n^{BDF4}	Plantation Area Data (ha)
0	2017		12,383,101	12,383,101
1	2018		13,057,634.1096861	14,326,350
2	2019		13,702,809.2846490	14,456,612
3	2020		14,314,134.6654177	14,586,597
4	2021	14,888,243.2170596	14,888,253.1720187	14,621,693
5	2022	15,422,947.8596378	15,422,932.9333459	15,338,556
6	2023	15,917,059.5759334	15,917,013.5988660	15,928,712
7	2024	16,370,382.8922089	16,370,301.3720848	16,013,039
8	2025	16,783,546.4880020	16,783,427.6853721	
9	2026	17,157,840.6636662	17,157,692.3351440	
10	2027	17,495,070.5515236	17,494,907.1738303	
11	2028	17,797,413.8486259	17,797,250.0736437	
12	2029	18,067,288.0006937	18,067,134.6721900	
13	2030	18,307,235.3664067	18,307,099.0074411	
14	2031	18,519,830.4845348	18,519,714.3449547	
15	2032	18,707,608.7696813	18,707,513.8675356	

Visualization of Numerical Solutions and Actual Data

To further evaluate the performance of the numerical schemes employed in this study, a visual comparison between the numerical solutions generated by the proposed methods and the actual oil palm plantation area data in Indonesia is presented. The goal of this visualization is to offer a visual evaluation of how closely the numerical estimates match the historical facts. The statistics show how well the model projections and the empirical data match, especially when it comes to capturing the general development pattern of the area under oil palm plantations. Furthermore, the results of the long-term projection are shown in order to analyze how the numerical solutions behave over time and to see how they tend to approach the carrying capacity specified in the Verhulst model. This graphical depiction makes it easy to assess each numerical scheme's consistency and dependability in explaining both long-term growth dynamics and short-term accuracy.

1. Comparison of AB4–BDF4 Numerical Results with Actual Data

Figure 2 shows a comparison between the numerical solution obtained using the AB4–BDF4 predictor–corrector method and the actual oil palm plantation area data in Indonesia. The numerical results closely follow the observed data during the available observation period, indicating good agreement between the model and real conditions. Furthermore, the projection illustrates a continuous increasing trend in plantation area, gradually approaching the assumed carrying capacity of 20 million hectares, which represents the upper limit of land availability in the Verhulst growth model. The close alignment is evident throughout the simulation, with relative errors remaining on the order of 10^{-6} (averaging 3.2477330×10^{-6}). The error is smallest in the early years (9.1389220×10^{-7} at $n = 4$) and peaks around $n = 9$ (5.0204146×10^{-6}) before gradually declining again toward the end of the observation period, indicating that the method's accuracy does not deteriorate over longer simulation horizons.

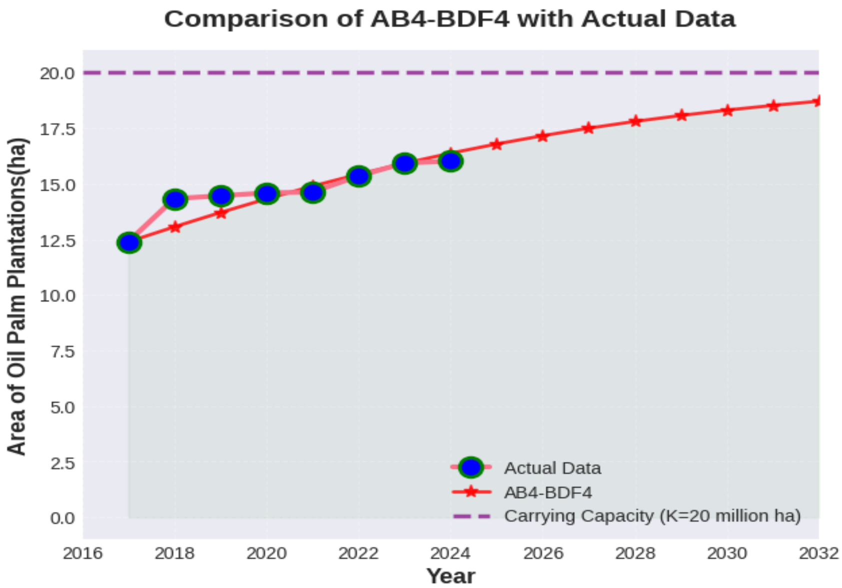

Figure 2. Comparison of AB4–BDF4 Numerical Results with Actual Data

2. Comparison of Milne–BDF4 Numerical Results with Actual Data

Figure 3 presents the comparison between the numerical results obtained using the Milne–BDF4 predictor–corrector scheme and the actual oil palm plantation area data in Indonesia. The Milne–BDF4 solution demonstrates a close correspondence with the observed data in the initial years, indicating that the method is capable of accurately capturing the growth pattern of plantation area. The projected trajectory shows a steady increase over time and gradually approaches the carrying capacity of 20 million hectares, reflecting the limiting effect incorporated in the Verhulst model. The relative error for the Milne–BDF4 scheme remains on the order of 10^{-6} throughout the simulation, averaging 5.9206896×10^{-6}. The error is smallest at $n = 4$ (6.6864520×10^{-7}) and rises to its highest point at $n = 10$ (9.3385859×10^{-6}), before gradually declining toward the end of the observation period — a trajectory similar to that of the AB4–BDF4 scheme, though with a later peak and consistently higher error values.

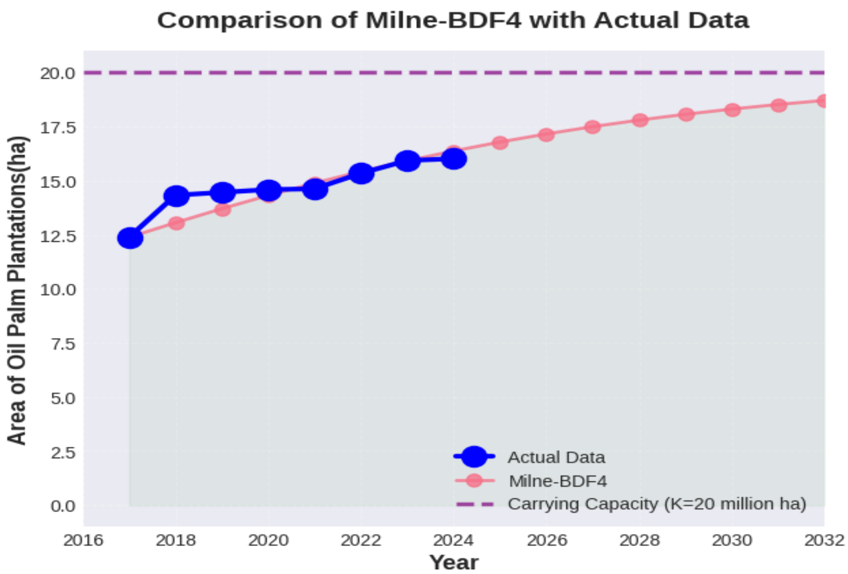

Figure 3. Comparison of Milne–BDF4 Numerical Results with Actual Data

3. Relative Error Comparison of AB4–BDF4 and Milne–BDF4

Figure 4 compares the relative errors of the AB4–BDF4 and Milne–BDF4 predictor–corrector schemes across different time intervals n on a logarithmic scale. Both schemes maintain modest errors within the range of 10^{-6} to 10^{-5}, but AB4–BDF4 consistently performs better, averaging 3.2477330×10^{-6} against 5.9206896×10^{-6} for Milne–BDF4 — approximately 1.8 times more accurate overall. Although both curves follow a similar pattern, rising through the middle of the simulation before tapering off near $n = 15$, the Milne–BDF4 curve remains consistently higher from $n = 6$ onward, indicating superior long-term stability for the AB4–BDF4 scheme.

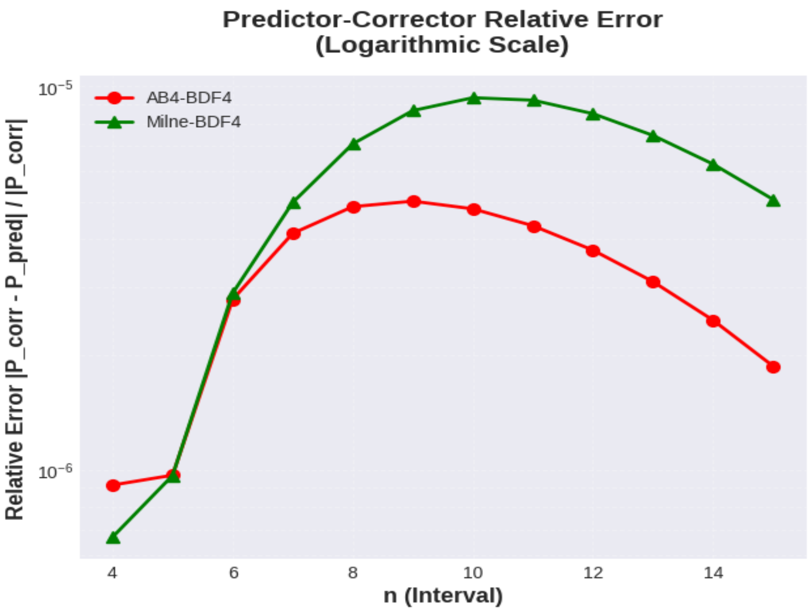

Figure 4. Relative Error Comparison of AB4–BDF4 and Milne–BDF4

DISCUSSION

The results of this study demonstrate that the Verhulst growth model is capable of describing the growth pattern of oil palm plantation area in Indonesia, which exhibits nonlinear behavior with a tendency toward saturation. The exponential model was ruled out early since it assumes unlimited growth, at odds with land bound by availability and policy restrictions. The Gompertz model was also considered, but its asymmetric saturation curve does not match the more balanced slowdown seen in the data — making the symmetric, capacity-driven behavior of the Verhulst model the more natural fit. The AB4–BDF4 and Milne–BDF4 predictor–corrector schemes yield numerical solutions that closely match the observed data, suggesting that both approaches are appropriate for numerically solving the Verhulst model. This result confirms the theoretical hypothesis that growth processes subject to economic, policy, and environmental constraints are best represented by logistic-type models (Anggreini et al., 2019; Susanti & Maryudi, 2016).

On average, the AB4–BDF4 scheme produces a relative error of 3.2477330×10^{-6}, compared to 5.9206896×10^{-6} for the Milne–BDF4 scheme — approximately 1.8 times more accurate overall. Notably, this advantage is not uniform across the simulation: at the earliest steps ($n = 4$ and $n = 5$), the Milne–BDF4 method performs marginally better, but from n = 6 onward, AB4–BDF4 consistently yields smaller errors, with the gap widening as the simulation progresses. This pattern is consistent with the theoretical behavior of the two predictors: AB4 draws on several previously computed values to estimate the next step, keeping its local truncation error small (on the order of h^5) and its accumulation more controlled over long intervals, whereas the Milne predictor, despite comparable order, relies on an explicit open formula that is more prone to error accumulation as the number of steps increases (Audu et al., 2022; Ibrahim et al., 2024; Romadhon, 2025; Suryani et al., 2023). Beyond accuracy, the bounded and non-divergent behavior of the relative error across all simulated intervals — rather than increasing indefinitely — indicates that both schemes remain numerically stable over the simulation period, with no evidence of error accumulation leading to divergence.

From a mathematical perspective, this study contributes to the application of numerical methods for solving nonlinear differential equation models in real-world contexts. The findings demonstrate the usefulness of predictor–corrector schemes in assessing numerical accuracy using relative error analysis and analyzing growth dynamics (Olatunji & Akeju, 2025; Tukan et al., 2024). This study offers a tangible illustration of how abstract numerical techniques can be applied to actual data in the context of mathematics education, giving them greater significance for teaching numerical analysis and applied mathematics (Raming et al., 2024). In practical terms, these findings offer a useful reference point for regional planning agencies seeking to anticipate how much additional land oil palm expansion may realistically require, helping balance agricultural development goals against land-use and conservation priorities.

Despite these contributions, this study has several limitations. The model does not specifically account for external influences such as policy changes, technological breakthroughs, or climate variability, and instead assumes constant growth parameters (Hidayat & Fajri, 2026; Mahtta et al., 2022). Additionally, the analysis relies on a relatively short observation period of eight annual data points, which may limit the statistical robustness of the parameter estimates. The intrinsic growth rate r was also estimated from a single year-pair interval rather than averaged across the full dataset, a choice that, while methodologically justified to avoid saturation bias, may not fully capture year-to-year variability in growth conditions.

Future research could address these limitations in several ways. To account for the influence of external factors, the model could be extended by incorporating time-dependent parameters or by combining numerical techniques with data-driven strategies, such as machine learning, capable of adapting to policy and climate variability (Okarda et al., 2024; Robbaniyyah et al., 2025). To address the limited data length, future studies could re-estimate the growth rate using a longer time series as more annual observations become available, allowing for a more robust, averaged estimation of r. Incorporating a time-varying carrying capacity would also better reflect shifting land-use policy over time, while validating plantation area estimates against independent sources, such as remote sensing data, could further strengthen the reliability of the model beyond official statistics.

CONCLUSION

The findings demonstrate that both the AB4–BDF4 and Milne–BDF4 approaches offer precise numerical approximations, with average relative errors of 3.2477330×10^{-6} and 5.9206896×10^{-6}, respectively. This corresponds to AB4–BDF4 being approximately 1.8 times more accurate than Milne–BDF4, particularly over longer simulation intervals, while both schemes remain numerically stable throughout the simulation period. The estimated intrinsic growth rate ($r \approx 0.15$) and carrying capacity ($K = 20$ million hectares) together characterize the long-term expansion behavior of oil palm plantation area in Indonesia. These results demonstrate the effectiveness of higher-order predictor–corrector techniques in resolving nonlinear growth models. From a practical standpoint, the findings could help with long-term agricultural planning by shedding light on potential growth patterns in relation to carrying capacity.

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INFORMED CONSENT

The authors have obtained informed consent from all participants.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

REFERENCE

Abanum, G. C. (2020). Comparison Of Numerical Methods In Predicting The Growth Of Normal Agricultural Assets. *Journal of Mathematical Sciences & Computational Mathematics*, *1*(4), 502–523. https://doi.org/10.15864/jmscm.1408

Adebisi, A. F., Aremu, S., Ogunniran, M. O., Tijani, K., & Ajileye, A. M. (2025). Enhanced Milne-Simpson's Methods For Autonomous And Singular Differential Equations. *Mathematics and Computational Sciences*, *6*(2), 125–140. https://doi.org/10.30511/mcs.2025.2035400.1205

Anggreini, D., Afifah, D. S. N., Hayuhantika, D., Hastari, R. C., & Puspasari, R. (2019). Population Projection Using The Implementation Of Differential Equation Of Logistic Models. *ICBLP 2019: Proceedings of the 1st International Conference on Business, Law And Pedagogy, ICBLP 2019, 13-15 February 2019, Sidoarjo, Indonesia, 326*. https://doi.org/10.4108/eai.13-2-2019.2286525

Anie, N. I. (2016). On The Accuracy Of The Backward Difference Formula Of Order Four And Runge-Kutta Of Order Four In Solving A First Order Non-Linear Ordinary Differential Equation. Kampala International University.

Audu, K. J., Garba, J., Tiamiyu, A. T., & Thomas, B. A. (2022). Application of Backward Differentiation Formula on Fourth-Order Differential Equations. *Journal of Science and Technology*, *14*(2), 52–65. https://doi.org/10.30880/jst.2022.14.02.006

BPS Indonesia. (2025). *Statistik Tanaman Perkebunan Tahunan Indonesia 2024 (Kelapa Sawit, Kopi, Kakao, Karet, Teh, dan Komoditas Perkebunan Unggulan)*. Bps.Go.Id. https://www.bps.go.id/id/publication/2025/08/29/8d2a6ab3510f9828daf73191/statistik-tanaman-perkebunan-tahunan-indonesia-2024--kelapa-sawit--kopi--kakao--karet--teh--dan-komoditas-perkebunan-unggulan-.html

Hidayat, T., & Fajri, S. (2026). Analysis of Public Policies in Indonesia's Palm Oil Industry to Support Sustainable Development. *ARISTO*, *14*(2), 998–1020. https://doi.org/10.24269/ars.v14i2.12130

Ibrahim, Z. B., Ijam, H. M., Aksah, S. J., & Abd Rasid, N. (2024). Enhancing accuracy and efficiency in stiff ODE integration using variable step diagonal BBDF approaches. *Malaysian Journal of Fundamental and Applied Sciences*, *20*(5), 1083–1100. https://doi.org/10.11113/mjfas.v20n5.3530

Isra, M., Sitepu, A. C., Panjaitan, A. I. A., Juniarta, M. R., Safitri, N., Hasibuan, U. B., & Harahap, V. W. L. (2025). Masa Depan Industri Kelapa Sawit Tantangan dan Peluang. *Jurnal Riset Rumpun*

Ilmu Tanaman, *4*(1), 275–282. https://doi.org/10.55606/jurrit.v4i1.5673

Mahtta, R., Fragkias, M., Güneralp, B., Mahendra, A., Reba, M., Wentz, E. A., & Seto, K. C. (2022). Urban land expansion: the role of population and economic growth for 300+ cities. *Npj Urban Sustainability*, *2*(1), 5. https://doi.org/10.1038/s42949-022-00048-y

Maulidya, A., Damayanti, A., Indra, T. L., & Dimyati, M. (2021). Prediction of Land Change for Oil Palm Plantations in Penajam Subdistrict, Penajam Paser Utara Regency, East Kalimantan Province. *Journal of Physics: Conference Series*, *1811*(1), 12072. https://doi.org/10.1088/1742-6596/1811/1/012072

Okarda, B., Purnomo, H., Juniyanti, L., Kusumadewi, S. D., & Nadhira, S. (2024). Indonesian palm oil towards sustainability: a system dynamic approach. *IOP Conference Series: Earth and Environmental Science*, *1379*(1), 12037. https://doi.org/10.1088/1755-1315/1379/1/012037

Olatunji, O., & Akeju, A. (2025). Comparative Analysis of Euler and Order Four Runge-Kutta Methods in Adams-Bashforth-Moulton Predictor-Corrector Method. *International Journal of Mathematics, Statistics, and Computer Science*, *3*, 276–293. https://doi.org/10.59543/ijmscs.v3i.10471

Raming, I., Wirawan, A. S., Putri, A. A., Sahputra, D. R., Dala, M. A. D., & Avrilia, M. P. (2024). Pemetaan pertumbuhan penduduk di Kota Samarinda melalui pemodelan logistik dengan metode Adams-Bashforth-Moulton. *Journal of Mathematics, Computations and Statistics*, *7*(1), 133–143. https://doi.org/10.35580/jmathcos.v7i1.1945

Robbaniyyah, N. A., Hanani, H., Ihwani, I. L., Awalushaumi, L., & Romdhini, M. U. (2025). Numerical Estimation of Rice Harvest Growth in West Lombok regency Using Adam-Bashforth-Moulton Method. *IOP Conference Series: Earth and Environmental Science*, *1493*(1), 12010. https://doi.org/10.1088/1755-1315/1493/1/012010

Romadhon, R. (2025). *Prediksi Produksi Daging Ayam Kampung di Provinsi Lampung Menggunakan Metode Adams-Bashforth-Moulton dan Metode Milne-Simpson*. Fakultas Matematika dan Ilmu Pengetahuan Alam, Universitas Lampung.

Shahputra, M. A., & Zen, Z. (2018). Positive and negative impacts of oil palm expansion in Indonesia and the prospect to achieve sustainable palm oil. *IOP Conference Series: Earth and Environmental Science*, *122*, 12008. https://doi.org/10.1088/1755-1315/122/1/012008

Suryani, I., Suprianto, A., & Wartono, W. (2023). Solusi Numerik Model Verhulst Pada Estimasi Pertumbuhan Produksi Padi Menggunakan Metode Milne-Simpson dan Metode Adams-Bashforth-Moulton. *Jurnal Sains Matematika Dan Statistika*, *9*(1), 27–36. https://doi.org/10.24014/jsms.v9i1.19694

Susanti, A., & Maryudi, A. (2016). Development narratives, notions of forest crisis, and boom of oil palm plantations in Indonesia. *Forest Policy and Economics*, *73*, 130–139. https://doi.org/10.1016/j.forpol.2016.09.009

Tukan, H., Maure, O. P., & Ina, K. T. D. (2024). Analisis Keakuratan Metode Numerik Dalam Menyelesaikan Turunan Persamaan Nonlinier. *Leibniz: Jurnal Matematika*, *4*(2), 10–22. https://doi.org/10.59632/leibniz.v4i02.446