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OPTIMAL CONTROL IN AN SDC MATHEMATICAL MODEL FOR DIABETES COMPLICATIONS AT A HOSPITAL IN LAMONGAN REGENCY

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ABSTRACT

Diabetes is a chronic disease whose prevalence continues to increase and has the potential to cause serious complications if not properly managed. This study aims to analyze the dynamics of diabetes progression and its complications, as well as to determine optimal control strategies using a mathematical model based on data from a hospital in Lamongan Regency. The model used is a compartmental SDC (Susceptible–Diabetes–Complication) model formulated as a system of ordinary differential equations with two time-dependent control variables. The optimal control is determined using Pontryagin's Maximum Principle, while the numerical simulations are solved using the fourth-order Runge–Kutta (RK4) method. Model parameters are obtained from the literature and epidemiological data, and then calibrated to match the characteristics of real-world cases. The simulation results show that without control, the susceptible population decreases from approximately 1500 to about 200 individuals, while the complication population increases to around 1700 individuals. With the implementation of optimal control, the susceptible population increases to approximately 1250 individuals, the number of diabetic patients decreases to around 820 individuals, and the complication population is reduced to about 980 individuals. These results indicate that control strategies focused on diabetic patients are effective in suppressing disease progression and preventing complications, and contribute to the development of data-driven mathematical models for local healthcare policy planning.

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INTRODUCTION

Diabetes is a chronic disease that has become one of the major global health problems. This disease is characterized by high blood glucose levels due to impaired insulin production or function, and in the long term can cause serious damage to various organs such as the heart, kidneys, nerves, and eyes (WHO, 2023). In addition to reducing patients' quality of life, diabetes also imposes a significant economic burden on global healthcare systems (Elmusharaf et al., 2025).

The increasing number of diabetes cases is influenced by various factors, including high-sugar and high-fat diets, lack of physical activity, obesity, as well as genetic and environmental factors (Kesehatan, 2018). Delayed diagnosis and low adherence to therapy can worsen patients' conditions and lead to various complications such as diabetic nephropathy, neuropathy, retinopathy, and cardiovascular disorders, which contribute to increased morbidity and mortality among diabetes patients (Trihono, 2023). According to the International Diabetes Federation, approximately 537 million adults worldwide were living with diabetes in 2021, and this number is projected to increase to 643 million by 2030 and 783 million by 2045. In Indonesia, the prevalence of diabetes in the adult population reaches 11.3% (IDF, 2025).

The development of mathematical science has significantly contributed to understanding disease dynamics through mathematical modeling based on systems of differential equations (Appadu et al., 2024). Mathematical models allow systematic analysis of changes in patient populations and evaluation of disease control strategies through numerical simulations (Kouidere et al., 2021). One applicable model is the SDC (Susceptible–Diabetes–Complication) model, which describes the transition of individuals from the susceptible population to diabetic patients and subsequently to the complication stage, based on recent developments in chronic disease modeling in the literature (Kouidere et al., 2020).

Research by Kouidere et al. (2020) in the article entitled "A New Mathematical Modeling with Optimal Control Strategy for the Dynamics of Population of Diabetics and Its Complications with Effect of Behavioral Factors" shows that increasing public awareness and early treatment can significantly reduce the number of diabetic patients through optimal control strategies that consider behavioral factors and public health campaigns. The model uses an optimal control approach to evaluate the effectiveness of behavioral interventions in controlling the dynamics of diabetic populations and their complications.

However, most diabetes models developed using optimal control approaches still have several limitations. Some previous models separate major risk factors, such as genetic and lifestyle factors, into different compartments, resulting in increased system complexity and making it difficult to directly interpret disease dynamics (Rohmah et al., 2022). In addition, some studies focus only on a single type of intervention (e.g., education or treatment alone), thus failing to provide a comprehensive picture of combined intervention strategies in controlling the progression of diabetes and its complications (Rohmah & Rahmalia, 2021). Furthermore, several models have not explicitly integrated the relationship between diabetes progression and complication stages within a unified dynamic system framework to support more applicable health policy analysis (Rohmah et al., 2025).

Based on these limitations, this study proposes a modified mathematical model of diabetes within an optimal control framework. The main novelty of this research is the integration of genetic and lifestyle risk factors into a single susceptible compartment (S), in contrast to Kouidere et al. (2020), who separated these factors into multiple risk compartments. This approach results in a simpler model while still being able to realistically represent the dynamics of diabetes progression.

In addition, this study integrates the modified SDC model with an optimal control approach using the Pontryagin Maximum Principle to determine optimal intervention strategies for controlling the progression of diabetes and its complications through two control variables, namely treatment for diabetic patients and management of patients with complications.

The contribution of this research lies not only in the development of the modified SDC model, but also in providing quantitative insights for decision-making in the healthcare sector by identifying the most effective interventions in reducing the progression of diabetes and its complications over time. The results of this study are expected to support more efficient and targeted allocation of healthcare resources.

METHOD

This study analyzes the dynamics of diabetes progression and its complications using a compartmental model based on a system of ordinary differential equations, along with an optimal control approach using Pontryagin's Maximum Principle.

The stages of this research are as follows:

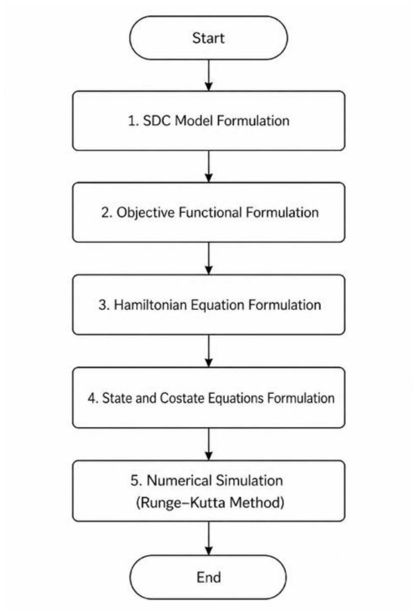

Figure 1. Research Flowchart

SDC Model Formulation

The SDC model is used to describe the dynamics of diabetes progression within a population. The population is divided into three compartments, namely susceptible individuals (S), diabetic patients (D), and diabetic patients with complications (C). The transitions between compartments are influenced by several parameters, such as the rate of diabetes progression, the rate of complication development, and the mortality rate. The model is then expressed in the form of a system of differential equations (Giacomelli & Passalacqua, 2021).

The mathematical model of diabetes with complications is formulated to describe the natural progression of the disease within a population in the absence of intervention. The total population is divided into three compartments, namely susceptible individuals S, diabetic individuals D, and individuals with complications C. The flow diagram of the uncontrolled model is presented in Figure 1.

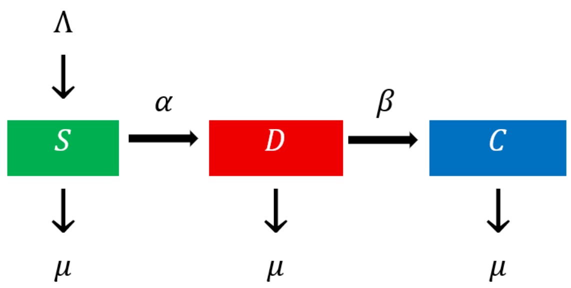

Figure 2. Flow Diagram of SDC Model Without Control

Based on the model assumptions, the system of differential equations is given as:

$$\frac{dS}{dt} = \Lambda - \beta S - \mu S \tag{1}$$
$$\frac{dD}{dt} = \beta S - \alpha D - \mu D \tag{2}$$
$$\frac{dC}{dt} = \alpha D - \mu C \tag{3}$$

Systems (6), (7), and (8) form the basis for developing the optimal control model.

To reduce the number of diabetic and complication cases, two time-dependent control variables are introduced: $u_1(t)$, representing the intensity of treatment for diabetic patients, and $u_2(t)$, representing the intensity of medical care for patients with complications (Wang et al., 2020).

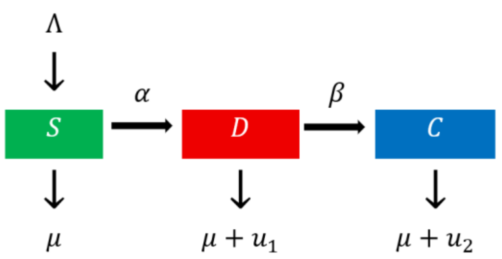

Figure 3. Flow Diagram of the SDC Model with Control

By incorporating the control variables, the system becomes:

$$\frac{dS}{dt} = \Lambda - \beta S - \mu S - u_1 D + u_2 C \tag{4}$$
$$\frac{dD}{dt} = \beta S - \alpha D - \mu D - u_1 D \tag{5}$$
$$\frac{dC}{dt} = \alpha D - \mu C - u_2 C \tag{6}$$

Initial conditions are given as: $S(0) = S_0,\ D(0) = D_0,\ C(0) = \ C_0$ represent the population sizes at a time $t = 0$. Notation:

$S(t)$	: Susceptible population
$D(t)$	: Diabetes patients
$C(t)$	: Patients with complications
Λ	: Recruitment rate into the population
β	: Diabetes progression rate
α	: Complication onset rate

μ	: Mortality rate
$u_1(t)$	: Control for diabetes
$u_2(t)$	: Control for complications

Control $u_1(t)$ functions to reduce the number of diabetes patients, while $u_2(t)$ reduces the number of complication cases.

Objective Functional Formulation

In optimal control problems, the objective functional is used to determine a control strategy that minimizes or maximizes the performance of a system over a given time interval (Grigorieva, 2021). In general, the objective functional is expressed as:

$$J(u) = \int_0^T L(x(t), u(t), t)dt + \Phi(x(T)) \tag{7}$$

$x(t)$ denotes the state variable, $u(t)$ denotes the control variable, $L(x(t), u(t), t)$ represents the running cost, and $\Phi(x(T))$ represents the terminal cost (Diveev et al., 2021).

The purpose of optimal control is to minimize the number of diabetes and complication cases while considering intervention costs. The instantaneous cost function is:

$$f(t) = A_1D(t) + A_2C(t) + \frac{1}{2}B_1u_1^2(t) + \frac{1}{2}B_2u_2^2(t) \tag{8}$$

Integrating over the interval $[0, T]$ gives the objective function:

$$J(u_1, u_2) = \int_0^T \left[A_1D(t) + A_2C(t) + \frac{1}{2}B_1u_1^2(t) + \frac{1}{2}B_2u_2^2(t)\right] dt \tag{9}$$

The controls $u_1(t)$ and $u_2(t)$ are chosen to minimize $J(u_1, u_2)$ subject to the state system constraints (Aye et al., 2025).

Hamiltonian Equation Formulation

The Hamiltonian function in optimal control problems is used to combine the objective functional with the system dynamics, enabling the determination of optimal conditions for the control variables (Shao et al., 2022). According to Lev S. Pontryagin, the Hamiltonian function can generally be expressed as follows:

$$H(x, u, \lambda, t) = L(x, u, t) + \lambda f(x, u, t) \tag{10}$$

$L(x, u, t)$ represents the running cost function, $f(x, u, t)$ represents the system dynamics function, and λ is the costate variable. Hamiltonian formulation is used in Pontryagin's Maximum Principle to obtain the characteristics of optimal control in a given system (Ndaïrou & Torres, 2023).

The Hamiltonian function of the obtained optimal control model can be written as:

$$H = \left(A_1 D + A_2 C + \frac{1}{2} B_1 u_1^2 + \frac{1}{2} B_2 u_2^2\right) + \lambda_S(\Lambda - \beta S - \mu S - u_1 D + u_2 C) + \lambda_D(\beta S - \alpha D - \mu D - u_1 D) + \lambda_C(\alpha D - \mu C - u_2 C) \quad (11)$$

According to Pontryagin's Maximum Principle, the optimal controls u_1^* and u_2^* must satisfy the stationarity conditions, meaning that the partial derivatives of the Hamiltonian with respect to the controls are equal to zero (Gutema et al., 2024):

$$\frac{\partial H}{\partial u_1} = 0 \qquad dan \quad \frac{\partial H}{\partial u_2} = 0$$

These conditions ensure that the Hamiltonian attains an optimal point with respect to the control variables.

For u_1:

$$\frac{\partial H}{\partial u_1} = B_1 u_1 - (\lambda_S + \lambda_D) D = 0 \quad (12)$$

Thus, the optimal control is given by:

$$u_1^* = \frac{(\lambda_S + \lambda_D) D}{B_1} \quad (13)$$

The model parameters are determined based on a combination of values obtained from relevant literature and a parameter adjustment process using real clinical data from a hospital in Lamongan Regency, Indonesia. Specifically, parameters such as the natural death rate and disease progression rates are adopted from published studies, while the transmission and complication rates are estimated through an adjustment process to fit the observed patient data. This process is carried out by minimizing the difference between the model simulation results and the actual clinical data, ensuring that the model can more accurately represent real-world diabetes progression and complications, thereby improving the reliability and applicability of the results.

As the number of diabetic individuals D increases, stronger intervention is required. The costate variables $(\lambda_S + \lambda_D) D$ represent the future value of the susceptible and diabetic populations. The weight parameter B_1 indicates that higher intervention costs lead to lower optimal control levels, ensuring efficient resource utilization (Leiva, 2025).

For u_2:

$$\frac{\partial H}{\partial u_2} = B_2 u_2 - (\lambda_S + \lambda_C) C = 0 \quad (14)$$

Thus, the optimal control is given by:

$$u_2^* = \frac{(\lambda_C - \lambda_S) C}{B_2} \quad (15)$$

As the number of complications C increases, a higher level of control is required. The costate difference $(\lambda_C - \lambda_S) C$ reflects the urgency of reducing complications or transitioning individuals toward lower-risk states. The weight parameter B_2 indicates that higher control costs reduce intervention intensity (You & Zhang, 2025).

In practice, the controls must remain non-negative and cannot exceed their maximum allowable values:

$$0 \leq u_1 \leq u_{1,max}, 0 \leq u_2 \leq u_{2,max}$$

Thus, the full optimal control characterization is expressed as:

$$u_1^* = max\left(0, min\left(\frac{(\lambda_S + \lambda_D)D}{B_1}, u_{1,max}\right)\right) \tag{16}$$

$$u_2^* = max\left(0, min\left(\frac{(\lambda_C - \lambda_S)C}{B_2}, u_{2,max}\right)\right) \tag{17}$$

In general, the Hamiltonian function can be expressed as the sum of the objective functional and the combination of costate variables with the system dynamics:

$$H = A_1 D + A_2 C + \frac{1}{2} B_1 u_1^2 + \frac{1}{2} B_2 u_2^2 + \lambda_S f_S + \lambda_D f_D + \lambda_C f_C \tag{18}$$

f_S, f_D, and f_C represent the dynamical functions of each state variable in the model (Weston et al., 2024).

State and Costate Equations Formulation

The state equation describes the dynamics of changes in the state variables of the system over time. In this model, the state variables consist of the susceptible population S(t), diabetic patients D(t), and patients with complications C(t) (Raţiu & Minculete, 2022). In general, the state equation can be written as follows:

$$\frac{dx}{dt} = f(x, u, t) \tag{19}$$

The costate equations are obtained from the partial derivatives of the Hamiltonian with respect to the state variables. These equations determine the optimal conditions of the system and are expressed as:

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial x} \tag{20}$$

x denotes the state variable, u denotes the control variable, H represents the Hamiltonian function, and λ denotes the costate variable (Shao et al., 2022).

The state equations describe the dynamics of population changes in each compartment over time by considering the influence of control variables. In this model, there are three compartments, namely the susceptible population S(t), diabetic patients D(t), and diabetic patients with complications C(t). The model involves two control variables, u_1 representing diabetes management and u_2 representing complication control (Mollah & Biswas, 2023).

The state system is given as follows:

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda - \beta S(t) - \mu S(t) - u_1 D(t) + u_2 C(t) \\ \dfrac{dD(t)}{dt} = \beta S(t) - \alpha D(t) - \mu D(t) - u_1 D(t) \\ \dfrac{dC(t)}{dt} = \alpha D(t) - \mu C(t) - u_2 C(t) \end{cases} \tag{21}$$

The state system is supplemented with initial conditions:

$$S(0) = 0, \quad D(0) = 0, \quad C(0) = 0$$

The costate equations are then determined to obtain the characteristics of the optimal control (K. A. et al., 2025).

Numerical Simulation Using Python Google Colab

The fourth-order Runge–Kutta (RK4) method is a numerical technique used to obtain approximate solutions of ordinary differential equations with a high level of accuracy. This method employs four slope approximations at each iteration step to compute the next solution value (Workineh et al., 2024). In general, for a differential equation of the form $\frac{dy}{dt} = f(t, y)$, the fourth-order Runge–Kutta method is expressed as follows:

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{22}$$

With:

$$\begin{aligned} k_1 &= hf(t_n, y_n) \\ k_2 &= hf\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right) \\ k_3 &= hf\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right) \\ k_4 &= hf(t_n + h, y_n + k_3) \end{aligned}$$

h denotes the step size (Pacôme et al., 2025).

In this study, numerical simulations are conducted using Python on the Google Colab platform with the following numerical parameters: step size h=0.01, simulation horizon $t \in [0,100]$, and initial values determined based on the initial population conditions of the SDC model, namely S(0), D(0), and C(0), according to the initial conditions of the system. These parameters are used to ensure the stability and accuracy of the simulation results in describing the system dynamics.

Result Interpretation

Result interpretation involves explaining the meaning of the analysis or simulation outcomes in the study, demonstrating the relationship between these results and the research objectives.

RESULTS

The numerical simulation results are used to evaluate the dynamics of the SDC system under two scenarios, namely without control and with the implementation of optimal control, in order to observe the effect of interventions on the spread of diabetes and its complications.

Table 1. Parameter Without Control

Parameter	**Value**	**Source**
Λ (Recruitment rate into the population)	30 (individual/mounth)	Assumed / calibrated from clinical trend
β (Diabetes progression rate)	0.05 (1/mounth)	Based on literature (Kouidere et al., 2020)
α (Complication onset rate)	0.08 (1/mounh)	Based on literature (Mollah & Biswas, 2023)
μ (Mortality rate)	0.05 (1/mounth)	WHO (2023), adjusted

The parameter values indicate that the recruitment rate Λ = 30 individuals per month represents a constant increase in the population. The diabetes progression rate β = 0.05 per month means that approximately 5% of susceptible individuals may develop diabetes each month. The complication onset rate α = 0.08 per month indicates that about 8% of diabetic individuals may progress to complications within a month. Meanwhile, the natural mortality rate μ = 0.05 per month implies that around 5% of the population dies in each time period.

Table 2. Parameter With Control

Parameter	**Value**	**Source**
Λ (Recruitment rate into the population)	65 (individual/mounth)	Assumed / calibrated from clinical trend
β (Diabetes progression rate)	0.1 (1/mounth)	Based on literature (Kouidere et al., 2020)
α (Complication onset rate)	0.06 (1/mounh)	Based on literature (Mollah & Biswas, 2023)
μ (Mortality rate)	0.02 (1/mounth)	WHO (2023), adjusted

The parameter values indicate that the recruitment rate Λ = 65 individuals per month represents a higher population inflow compared to the uncontrolled case. The diabetes progression rate β = 0.1 per month means that approximately 10% of susceptible individuals may develop diabetes each month. The complication onset rate α = 0.06 per month indicates that about 6% of diabetic individuals may progress to complications within a month. Meanwhile, the natural mortality rate μ = 0.02 per month implies that around 2% of the population dies in each time period.

Numerical simulations are conducted to analyze the dynamics of the SDC model under two scenarios, namely without control and with control. The simulations are performed using the fourth-order Runge–Kutta (RK4) method to obtain approximate solutions of the system of differential equations. The results of the simulation without control are presented in Figure 3, while the results of the simulation with control are presented in Figure 4.

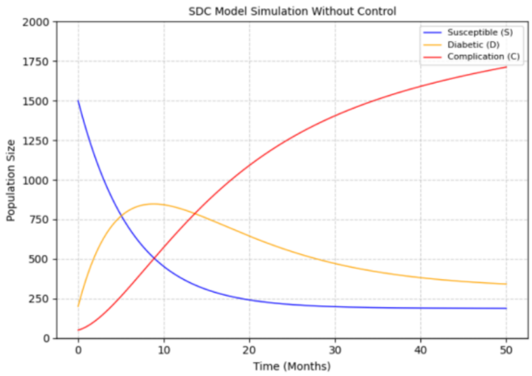

Figure 4. SDC Model Simulation Without Control

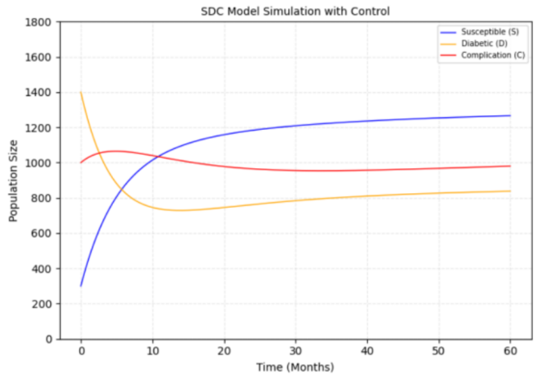

Figure 5. SDC Model Simulation With Control

In the simulation without control, the susceptible population decreases sharply from approximately 1500 individuals at the initial time to around 300 individuals by month 10, and then gradually declines to about 200 individuals by month 50. The diabetic population initially increases from about 200 individuals to a peak of approximately 800–900 individuals around months 8 to 10, and then gradually decreases to around 300 individuals by the end of the simulation. Meanwhile, the number of individuals with complications increases continuously from about 50 individuals at the beginning to more than 1700 individuals by month 50, indicating a significant rise in complications without intervention.

In contrast, in the simulation with control, the susceptible population increases from approximately 300 individuals at the initial time to around 1100 individuals by month 10, and continues to rise until it stabilizes at about 1250 individuals by month 60. The diabetic population decreases from around 1400 individuals to approximately 750 individuals by month 10, then slightly increases and stabilizes within the range of 800–850 individuals until the end of the simulation. Meanwhile, the number of individuals with complications remains relatively controlled, starting from around 1000 individuals, slightly increasing to about 1100 individuals around month 5, and then decreasing and stabilizing within the range of 950–1000 individuals toward the end of the simulation.

DISCUSSION

The numerical simulation results show a significant difference between the uncontrolled and controlled scenarios. In the simulation without control, the susceptible population decreases sharply from approximately 1500 individuals to around 300 individuals by month 10, while the number of complication cases increases significantly to more than 1700 individuals by the end of the simulation. In addition, the diabetic population initially rises to about 800–900 individuals before gradually declining. This pattern indicates that, without intervention, the disease progresses continuously from susceptible individuals to diabetes and eventually to complications, leading to a worsening system condition.

This phenomenon is influenced by the model parameters, particularly the diabetes progression rate $\beta = 0.05$ per month and the complication onset rate $\alpha = 0.08$ per month, which are relatively high and accelerate transitions between compartments. In real-life contexts, this condition can be associated with unhealthy lifestyles such as high consumption of sugar and fat, lack of physical activity, and sedentary behavior. In addition, genetic factors also contribute to increasing an individual's risk of developing diabetes. The absence of control in the model represents conditions where public awareness is low, early detection is limited, and medical treatment is not properly implemented, thereby accelerating disease progression toward complications.

In contrast, in the controlled simulation, the susceptible population increases from approximately 300 individuals to more than 1100 individuals by month 10 and stabilizes above 1200 individuals at the end of the simulation. The diabetic population decreases from around 1400 individuals to about 750 individuals in the early phase, then stabilizes at approximately 800 individuals. Meanwhile, complication cases are effectively controlled within the range of 950–1000 individuals throughout the simulation period. This indicates that the implementation of control significantly improves the system dynamics.

This improvement is influenced by changes in parameter values, namely the recruitment rate $\Lambda = 65$, the diabetes progression rate $\beta = 0.1$, the reduced complication rate $\alpha = 0.06$, and the mortality rate $\mu = 0.02$. The lower value of α indicates more effective control in preventing complications, while the lower μ reflects improved survival due to better medical treatment. In real-world terms, this condition can be interpreted as the result of interventions such as consistent medical treatment, improved access to healthcare services, early disease detection, and lifestyle changes including regular physical activity and healthier dietary patterns. With the presence of control, public awareness of maintaining health also increases, which helps suppress disease progression.

The main contribution of this study lies in the modified SDC model, which is capable of quantitatively describing the differences in system dynamics before and after control, as well as linking them to real-world interpretations. The model provides a relatively simple yet representative mathematical approach for analyzing chronic diseases such as diabetes and its complications.

However, this study has limitations, as some parameter values are still based on assumptions and calibration processes, and the model does not explicitly incorporate genetic and environmental factors. Therefore, future research is recommended to use more comprehensive data and develop more complex models to obtain more accurate and realistic results.

CONCLUSION

Based on the numerical simulation results of the SDC model, it can be concluded that the implementation of optimal control significantly improves the dynamics of diabetes and complication populations. In the absence of control, the susceptible population decreases from approximately 1,500 individuals to around 200 individuals, while complication cases increase to about 1,700 individuals, indicating uncontrolled disease progression. In contrast, with the application of optimal control, the susceptible population increases to approximately 1,250 individuals, and complication cases decrease to around 980 individuals, showing a more stable system. These results address the research gap by demonstrating that a simplified SDC model with optimal control can effectively capture and regulate the progression of diabetes and its complications. Therefore, focusing control at the diabetes stage is sufficient to suppress progression to complications. In practice, this implies that early intervention, regular treatment, and healthy lifestyle management are essential strategies to reduce complication risks and improve population health outcomes.

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included requests for translation, paraphrasing, and clarification of research explanations. The outputs generated from these prompts were used as supporting materials in the writing of the article. The author remains the sole author of this article and takes full responsibility for the entire content of the manuscript in accordance with COPE.

INFORMED CONSENT

The author declares that participant consent was not required, as this study did not involve human participants.

CONFLICT OF INTEREST

The author declares that there are no conflicts of interest in this study.

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