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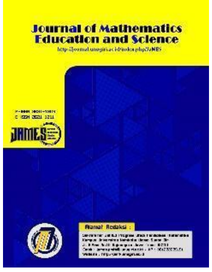

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STUDENTS' GEOMETRIC THINKING PROCESSES THROUGH THE LENS OF DUVAL'S AND VAN HIELE'S THEORIES: A GENDER-BASED ANALYSIS

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ABSTRACT

This study aims to explore the geometric thinking processes of junior high school students in solving problems involving flat-sided three-dimensional shapes, with a focus on gender differences. The analysis was conducted by integrating Van Hiele's theory of geometric development levels and Duval's cognitive theory. This was a qualitative study using a collective instrumental case study design. The research subjects consisted of 2 male and 2 female ninth-grade students at SMP Negeri 24 Penajam Paser Utara. The instruments used were the Van Hiele Geometry Test (VHGT) and problem-solving task-based interviews using the think-aloud method. The validity of the findings was assessed using triangulation. The results showed that Level 0 (visualization) students' thinking was heavily dependent on static visual representations; they experienced cognitive paralysis in the symbolic monofunctional register; and they failed in formal discursive apprehension. Level 1 (analysis) students were able to analytically break down functional geometric components, mastered the principle of spatial invariance, and were skilled at performing multidirectional conversions between semiotic registers. Female subjects tended to be formal, procedural, structured, and sequential, and were strong in verbal and written articulation. Male subjects tended to be globally intuitive through direct mental modeling but were prone to encountering obstacles in the realm of formal verbal mathematical discourse.

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INTRODUCTION

Geometry plays a fundamental role in mathematics education because it develops students' spatial visualization, logical reasoning, and problem-solving abilities that are essential for understanding mathematical concepts and solving real-world problems (Gantert, 2008; Hasratuddin, 2022). However, international assessments indicate that students' mathematical competence remains a major educational challenge. According to the OECD Programme for International Student Assessment (OECD, 2023), Indonesian students obtained an average mathematics score of 366, substantially below the OECD average of 472, and only 18% of students attained at least Level 2 proficiency in mathematics, indicating persistent difficulties in mathematical reasoning and problem solving. Consistent with these findings, previous studies have reported that junior high school students continue to experience difficulties in interpreting geometric representations, identifying the properties of three-dimensional objects, and solving non-routine geometry problems (Kusuma et al., 2021; Putri & Nurlaelah, 2022; Rahayu & Jupri, 2021). A similar condition was identified at SMP Negeri 24 Penajam Paser Utara. The results of the 2026 Academic Aptitude Test (TKA) showed that students' average mathematics score was only 34.81, with 11 students categorized as "adequate" and 7 as "below adequate." Furthermore, data from the 2025 Computer-Based National Assessment (ANBK) indicated that students' numeracy achievement remained at the moderate level, while the average score in the geometry domain declined by 5.65 points compared with the previous year.

Students face significant challenges in interpreting representations of three-dimensional shapes, identifying the intrinsic elements of shapes, and applying geometric concepts to solve non-routine problems. Junior high school students often make mistakes when determining surface area, calculating volume, and drawing net diagrams of flat-sided three-dimensional shapes (Putri & Nurlaelah, 2022). A decline in geometric performance calls for an in-depth evaluation using a relevant cognitive framework. Van Hiele’s levels of thinking and Duval’s cognitive model are two theories that underpin the geometric thinking process (Jones, 1998). Although these two theories have divergent research focuses, integrating them provides a comprehensive understanding of how students think.

Van Hiele's theory describes the hierarchical development of students' geometric thinking from visualization (Level 0), analysis (Level 1), informal deduction (Level 2), formal deduction (Level 3), to rigor (Level 4) (Usiskin, 1982). Previous studies consistently reported that most junior high school students remain at the visualization, analysis, or informal deduction levels, indicating that many students have not yet developed higher-order geometric reasoning (Kusuma et al., 2021; Nusaibah et al., 2021; Putri & Nurlaelah, 2022; Rahayu & Jupri, 2021). Although these studies successfully identified students' geometric thinking levels and learning difficulties, they mainly classified students according to Van Hiele's hierarchy without explaining how students construct geometric meaning through different forms of representation during problem solving. Consequently, students at the same Van Hiele level may still demonstrate different thinking processes when interpreting, transforming, and communicating geometric representations.

Duval's theory complements Van Hiele's framework by explaining how students construct mathematical meaning through semiotic representation registers, semiotic transformations, figure apprehension, and cognitive processes during geometry learning (Duval, 1988, 2006; Hershkowitz et al., 1998). Unlike Van Hiele's theory, which primarily classifies students according to hierarchical levels of geometric thinking, Duval's theory focuses on the cognitive mechanisms involved when students interpret, transform, and coordinate different mathematical representations during problem solving. Therefore, integrating these two theories enables a more comprehensive analysis of students' geometric thinking by combining hierarchical cognitive development with representational processes. However, only a limited number of studies have integrated these complementary perspectives to investigate students' geometric thinking processes, particularly in solving three-dimensional geometry problems from a gender perspective. This gap provides the rationale for the present study.

In addition to cognitive aspects, the gender variable also shows a significant correlation with and influence on the structure of mathematical thinking processes (Sudihartinih & Wahyudin, 2020). Gender differentiation encompasses cognitive variations in spatial perception and problem-solving (Palobo et al., 2024). Research in cognitive psychology shows a polarization between groups of males, who tend to rely on global spatial intuition and direct mental modeling, and groups of females, who demonstrate strengths in formal procedural regulation, sequential reasoning, and verbal articulation. This study aims to conduct an in-depth analysis of students' geometric thinking processes, as viewed through Duval and Van Hiele's theory, based on gender, using flat-sided three-dimensional shapes as the subject matter.

METHOD

This study employed a qualitative approach using a collective instrumental case study design (Stake, 1995). This design was selected because the study sought to obtain an in-depth understanding of students' geometric thinking processes rather than to generalize findings. By investigating four cases representing different Van Hiele thinking levels and gender, the study enabled cross-case comparisons to reveal similarities and differences in students' cognitive processes as interpreted through Van Hiele's and Duval's theoretical frameworks.

The study aimed to explore the geometric thinking processes of junior high school students in solving problems involving flat-sided three-dimensional shapes based on Van Hiele's levels of thinking, as viewed through the lens of gender differences. The study was conducted during the second semester of the 2025/2026 academic year in Grade 9 at SMP Negeri 24 Penajam Paser Utara. Data were collected through three stages: administration of the Van Hiele Geometry Test (VHGT), completion of the problem-solving task, and task-based think-aloud interviews with the selected participants. The Van Hiele Geometry Test (VHGT), developed as part of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project, consists of 25 multiple-choice items, with five items representing each Van Hiele thinking level (Usiskin, 1982). The instrument has been widely used in geometry education research and has demonstrated satisfactory validity and reliability for assessing students' levels of geometric thinking (Senk et al., 2022; Usiskin, 1982). Therefore, the VHGT was employed to classify students according to their Van Hiele thinking levels prior to selecting participants for the in-depth qualitative investigation.

Participants were selected using purposive sampling (Creswell, 2012). The selection was based on the students' results on the Van Hiele Geometry Test (VHGT), which was administered to all 18 ninth-grade students. Students who consistently met the classification criteria for the Visualization level (Level 0) and Analysis level (Level 1) were considered potential participants because these two levels were the focus of this study. To enable comparisons based on gender while maintaining an in-depth qualitative analysis, four students were selected, consisting of one male and one female student from Level 0 and one male and one female student from Level 1. In addition to their VHGT results, the selected participants voluntarily agreed to participate in the interviews and the entire research process (Creswell, 2012). For confidentiality, the participants were assigned the codes SL0 (male Level 0), SP0 (female Level 0), SL1 (male Level 1), and SP1 (female Level 1).

The main research instrument was a problem-solving task consisting of non-prototype flat-sided three-dimensional geometry problems designed to elicit students' geometric thinking processes. The tasks emphasized the deconstruction of complex three-dimensional figures, including cube nets, sketches of slanted prisms, and slanted prism containers. Prior to data collection, the problem-solving task was reviewed by two mathematics education experts. The validation assessed content relevance, language clarity, geometric accuracy, and the ability of the tasks to elicit students' geometric thinking processes. The instrument achieved an average validity score of 3.80 out of 4.00, indicating a high level of content validity. Based on the experts' feedback, minor revisions were made to improve the clarity of the instructions and the wording of several items before the instrument was administered. Following the completion of the written task, task-based think-aloud interviews were conducted with the selected participants to confirm their written responses, clarify their reasoning, and obtain a deeper understanding of their geometric thinking processes (Kusuma et al., 2021).

To ensure the trustworthiness of the findings, credibility was established through data triangulation between students' written responses and think-aloud interview data. Dependability was enhanced by documenting all research procedures and maintaining a systematic audit trail throughout the study. Confirmability was ensured through regular discussions among the researchers during the coding and interpretation processes. Transferability was supported by providing rich descriptions of the research context, participants, research instruments, and data collection procedures (Lincoln & Guba, 1985).

Data analysis was conducted iteratively throughout the study following five systematic steps. First, students' written responses and think-aloud interview recordings were transcribed and organized. Second, the data were coded using an integrated coding framework derived from Van Hiele's levels and Duval's theoretical constructs, including semiotic representation, semiotic transformation, and figure apprehension. Third, the coded data were analyzed within each case to identify the characteristics of individual students' geometric thinking processes. Fourth, a cross-case analysis was conducted to compare similarities and differences between male and female students within and across Van Hiele thinking levels. Finally, the findings were interpreted by integrating Van Hiele's hierarchical levels of geometric thinking with Duval's cognitive framework to provide a comprehensive explanation of students' geometric thinking processes (Creswell, 2012).

RESULTS

Based on the results of the grouping selection using the Van Hiele Geometry Test (VHGT) on 18 students, the distribution of students' thinking levels by gender is presented in Table 1 below.

Table 1. Distribution of Students' Van Hiele Thinking Levels by Gender

No.	Van Hiele Development Levels	Male	Female	Total
1.	Pre-Visualization Level	5	1	6
2.	Visualization Level (L0)	3	1	4
3.	Pre-Analysis Level	0	0	0
4.	Analysis Level (L1)	1	1	2
5.	Informal Pre-Deduction Level	0	0	0
6.	Informal Deduction Level (L2)	0	1	1
7.	Pre-Formal Deduction Level	0	0	0
8.	Formal Deduction Level (L3)	0	0	0
9.	Pre-Rigor Level	0	0	0
10.	Rigor Level (L4)	0	0	0
11.	The Jumping Phenomenon (No Fit)	4	1	5
	Total	**13**	**5**	**18**

The data in Table 1 show that students' geometry skills are still predominantly at the basic and pre-basic levels. Six students are at the Pre-Visualization level and four students are at the Visualization level (L0). Only two students have reached the Analysis level (L1), and one female student has reached the Informal Deduction level (L2). There is a unique phenomenon involving five students who fall into the "No Fit" category, or a "level leap" phenomenon, because the sequence of their cognitive responses does not follow Van Hiele's linear hierarchy. Overall, the distribution indicates that most students had not yet progressed beyond the early stages of Van Hiele's geometric thinking hierarchy, with the majority concentrated at the Pre-Visualization and Visualization levels. Therefore, the qualitative analysis focused on students at the Visualization (L0) and Analysis (L1) levels to examine differences in geometric thinking processes between male and female students.

The qualitative analysis focused on four participants representing both genders at the Visualization (L0) and Analysis (L1) levels, namely SL0, SP0, SL1, and SP1. These two levels were selected because both male and female students were represented, enabling meaningful gender-based comparisons within the same Van Hiele thinking level. Although one student reached the Informal Deduction level (L2), this level was represented by only one female student. Therefore, it was not included in the qualitative analysis because meaningful comparisons between male and female students at the same thinking level could not be conducted.

Analysis of Geometric Thinking Processes SL0

SL0 exhibits thinking characteristics at the visualization level that are static or passive in nature. SL0 relies purely on visual guesswork at a glance when asked to mentally simulate the folding of a series of cube nets. SL0 is stuck at the level of perceptual apprehension, selecting squares based on their relative spatial positions on the paper and is unable to mentally simulate the folding process. When asked about the reason for selecting a specific cube net, SL0 stated that the choice was based solely on visual appearance. During the interview, SL0 admitted to guessing the answer based on the shape ("I just looked at the picture") and expressed uncertainty regarding whether the net could actually be folded into a cube ("I'm not sure. I only guessed from the shape"). Furthermore, SL0 was unable to explain why the other options were invalid, noting that the images appeared virtually identical ("I don't know. They all look almost the same to me").

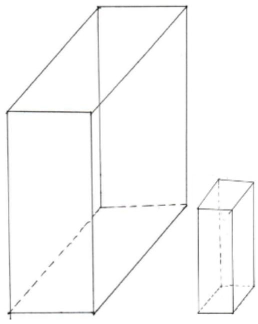
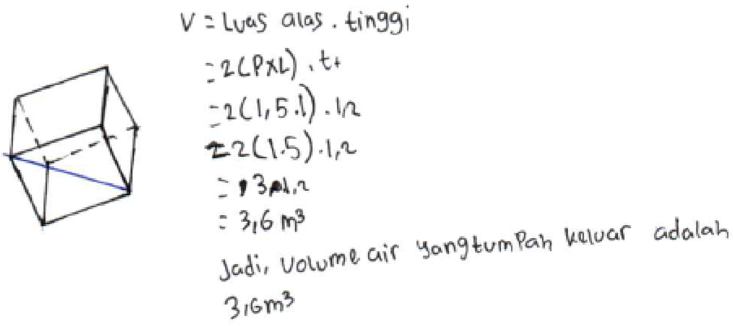


Figure 1. SL0 Answers to the Problem-Solving Assignment

Figure 1 shows that SL0 drew a sketch of a rectangular prism with dimensions that did not correspond to those provided in the problem (1.5 m × 1 m × 1.2 m). The drawing also did not correctly represent the position of the water surface after the container was tilted. During the interview, SL0 explained that the drawing was based on an initial impression of the figure rather than the given measurements, stating, “I just drew what I thought it looked like.” When asked how the water level would change after the container was tilted, SL0 responded, “I don't know because I couldn't imagine it.” These responses are consistent with SL0's written work, which shows difficulty in representing the geometric situation described in the problem.

Analysis of Geometric Thinking Processes SL1

SL1 demonstrated stronger cognitive performance at the analytical level. SL1 was able to determine the base of a cube through mental simulation of the net without physical aids. The uniqueness of SL1’s thought process was evident when they encountered an obstacle in component analysis due to visual distortion in a perspective image, yet successfully corrected it independently through discursive apprehension. SL1 was able to demonstrate the law of positional invariance, that tilting a rectangular prism does not change its volume, and to perform conversion transformations and algebraic operations on a symbolic monofunctional register with ease.



$$
\begin{aligned}
V &= Base\ Area. Height \\
&= 2(p \times l). t \\
&= 2(1{,}5.1). 1{,}2 \\
&= 2(1{,}5). 1{,}2 \\
&= 3.1{,}2 \\
&= 3{,}6\ m^3
\end{aligned}
$$

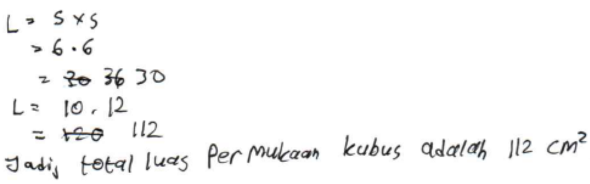
So, the volume of water that spilled out is 3,6 m^3

Figure 2. SL1 Answers to the Problem-Solving Assignment

Figure 2 shows that SL1 correctly represented the remaining water after the container was tilted and successfully calculated the remaining water volume. During the interview, however, SL1 experienced difficulty explaining the mathematical reasoning behind the triangular water surface. When asked why the remaining water formed a right-angled triangle, SL1 paused for a considerable time before responding, "I know the shape is like this, but I don't know how to explain why." When prompted further, SL1 added, "I can imagine the water after the container is tilted, but it's difficult to put it into words." These interview responses indicate that SL1 was able to mentally visualize the geometric configuration but experienced difficulty expressing the underlying mathematical reasoning verbally.

Analysis of Geometric Thinking Process SP0

SP0 exhibits cognitive characteristics at the visualization level that absolutely require the presence of concrete physical objects in order to reason. The physical act of folding pieces of paper serves as the sole cognitive bridge for SP0. A complete cognitive block occurs in the geometric reasoning process when moving into numerical problem-solving. A critical failure in the conversion process occurs when transitioning from the figural register of a sequence of six squares to the monofunctional symbolic register of algebraic or numerical formulas.



$$
\begin{aligned}
L &= s \times s \\
&= 6.6 \\
&= 30 \\
L &= 10.12 \\
&= 112
\end{aligned}
$$

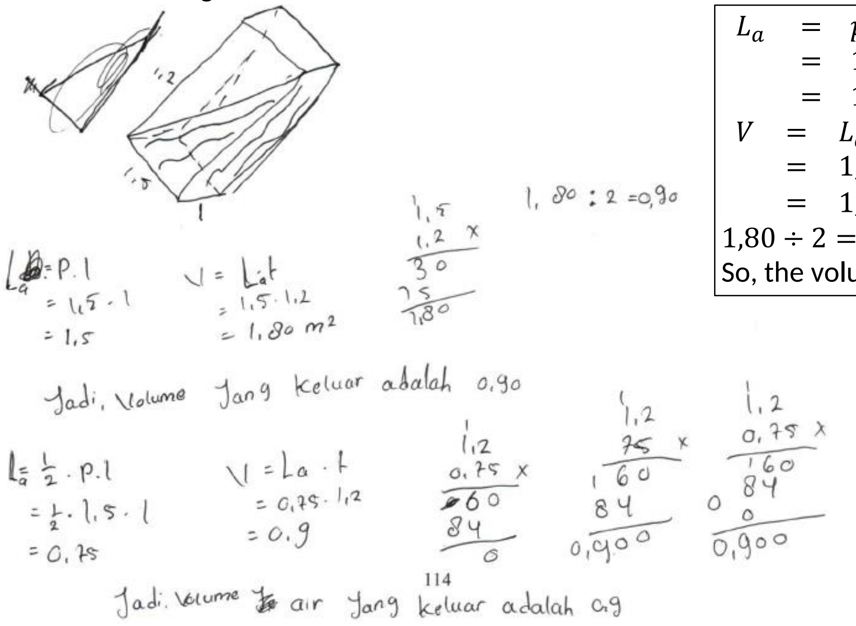
So, the total surface area of the cube is 112 cm^2

Figure 3. SP0 Answers for the Problem-Solving Assignment

Figure 3 shows that SP0 identified the faces of the cube as squares but used an incorrect formula by multiplying the side length by the total number of cube edges to determine the required quantity. During the interview, SP0 explained that the calculation was based on the assumption that all cube edges should be multiplied by the side length, stating, “I multiplied the side length by all the edges because the cube has twelve edges.” When asked why this formula was used instead of a volume formula, SP0 replied, “I thought every part of the cube should be counted first.” Furthermore, when discussing the tilted container, SP0 stated, “The water gathers at the bottom, so I think the volume changes.” These responses are consistent with SP0's written work, showing that the visual representation was not successfully translated into an appropriate mathematical expression.

Analysis of Geometric Thinking Process SP1

SP1 demonstrated geometric thinking skills that reflect a highly developed level of analysis. SP1 was able to perform high-level spatial abstraction through mental rotation without physical aids. SP1 demonstrated a very strong grasp of discursive apprehension by identifying nets as representations of the six congruent square faces that make up a cube. SP1 demonstrates a high degree of precision in sequential apprehension, labeling the vertices in a circular and sequential manner according to standard mathematical conventions.



$$
\begin{aligned}
L_a &= p.l \\
&= 1{,}5.1 \\
&= 1{,}5 \\
V &= L_a.t \\
&= 1{,}5.1{,}2 \\
&= 1{,}80\ m^2
\end{aligned}
$$

$1{,}80 \div 2 = 0{,}90$

So, the volume that came out is 0,90

$$
\begin{aligned}
L_a &= \frac{1}{2}.p.l \\
&= \frac{1}{2}.1{,}5.1 \\
&= 0{,}75 \\
V &= L_a.t \\
&= 0{,}75.1{,}2 \\
&= 0{,}9
\end{aligned}
$$

So, the volume of water that came out is 0,9

Figure 4. SP1 Answers for the Problem-Solving Assignment

Figure 4 shows that SP1 solved the problem using two different solution strategies. In the first strategy, SP1 calculated the volume of the entire rectangular prism and then divided the result by two to obtain the remaining water volume. In the second strategy, SP1 first determined the area of the triangular cross-section formed after the container was tilted and then multiplied it by the height of the container. During the interview, SP1 explained, “At first I calculated the whole volume and then took half of it because only half of the container was filled.” SP1 further stated, “Another way is to calculate the area of the triangle first and then multiply it by the height. Both methods give the same result.” These responses are consistent with SP1's written work, indicating that the student was able to construct and justify more than one valid solution strategy.

After the individual analyses of the written test and interview data, all findings were coded using Duval's theory of semiotic representation to identify patterns of geometric thinking across participants. The coding focused on the participants' use of figural, discursive, and symbolic representations, as well as their ability to transform information between these semiotic registers. The coded findings were then synthesized into the analytical categories presented in Table 2 to facilitate comparison among participants.

Table 2. Summary of Duval's Cognitive Activities in Solving Geometry Problems Involving Flat-Sided Three-Dimensional Shapes

Duval Components	Code	SL0	SL1	SP0	SP1
Semiotic Representation Register					
Multifunctional Register	Rmu	Very limited, passive, and dominated by "I don't know" responses. Random figurative sketches created solely for external aesthetic appeal	Able to draw proportional sketches, but was occasionally affected by the perspective distortion caused by the tilted 3D model	Able to use natural language in text, but verbal arguments are influenced by physical perception; a tilted position is considered more significant	Extremely strong, capable of constructing precise figural sketches and fluently articulating logical and mathematical verbal arguments
Monofunctional Register	RMo	Leave the answer sheet blank, no formulas or numbers written on it	Able to apply formal formulas mechanically and procedurally and perform numerical calculations correctly	Enable the symbolic register in random mode so that it contains random arithmetic operations that are conceptually incorrect	The execution of formal formulas and symbolic manipulation of decimal numbers is very neat, systematic, and accurate
Semiotic Transformation					
Treatment	TP	No algebraic or numerical operations are currently running	Able to perform numerical calculations smoothly	Experiencing an internal calculation error due to a misconception	Internal arithmetic operations are executed sequentially and with precision
Conversion	TK	A limitation that prevents dimensional measurements from being translated into drawings or formulas	Successfully converted to symbolic register, despite being temporarily hindered by visual bias	It is capable of the initial conversion of text to images, but fails completely when transitioning from the figurative to the symbolic register	Smooth multi-directional conversion of natural language → figural → symbolic
Types of Apprehension					
Perceptual Apprehension	AP	Focusing exclusively on outward appearances and mere	Momentarily distracted by the slanted projection lines in the 3D image	Trapped in a global shape and claiming a large area, but only the visible side is painted	Viewing a representation as a set of constant functional

Duval Components	Code	SL0	SL1	SP0	SP1
		visual guesswork			geometric properties
Sequential Apprehension	AS	Failure due to ignoring the order rules and dimensional constraints	Successfully reconstructing the inclined beam into a vertical one in a sequential manner	Failed due to zigzag and random vertex naming	Successfully labeling consecutive vertices of a circle according to standard conventions
Discursive Apprehension	AD	Completely paralyzed because the networks are isolated from the theoretical framework	Temporal obstacles due to difficulty articulating theoretical reasons at half volume	Unable to relate visualizations of figures to mathematical theorems or properties	It is very sturdy because it directly connects the nets using congruent 6-sided polygons
Operational Apprehension	AO	Unable to perform mental modifications or folding simulations	Using spatial intuition to modify one's perception of space	Requires physical modifications, such as cutting or folding the paper, to aid in reasoning	Ability to mentally rotate nets and deconstruct the space of an oblique block
Cognitive Processes in Geometry					
Visualization	V1 and V2	Relying exclusively on a static V1 and a non-functional V2 due to non-standard orientation	V2's capacity is strong enough to intuitively grasp the phenomenon of tilted containers	Using V1, but V2 fails when the object is tilted	High-level V2 capacity and the ability to consistently visualize changes in spatial position
Construction	K	Failed to mark the construction product as invalid due to ignoring the original dimensions	Successfully redrew the slanted block so that it stands upright while maintaining its proportions	Can construct a new sketch, even if the corner point symbols are misplaced	The construction of the vertical beam sketch is highly precise, complete with valid edges and vertices
Reasoning	P1, P2, P3, and P4	Complete paralysis in all aspects of reasoning	Capable of intuitive (P1) and operational reasoning, but has difficulty with discursive articulation (P2, P4)	Able to draw conclusions based only on physical perception (P1); failed on P2, P3, and P4	Demonstrates excellent mastery of P2 and P3, and has proven capable of presenting two formal solution paths (P4)

Overall, Table 2 shows a progressive development of geometric thinking across the four participants. Participants at the lower levels (SL0 and SP0) experienced greater difficulty coordinating figural, symbolic, and discursive representations, resulting in incomplete or incorrect mathematical reasoning. In contrast, participants at the higher levels (SL1 and SP1) demonstrated more effective use of multiple semiotic representations. Among them, SP1 exhibited the greatest flexibility by employing two different but mathematically equivalent solution strategies. These findings provide an overview of the participants' representational characteristics, which are discussed in relation to Duval's theory in the following section.

DISCUSSION

This study revealed clear differences in geometric thinking between students at Van Hiele Level 0 (Visualization) and Level 1 (Analysis), as well as differences associated with gender. Students at Level 0 tended to rely primarily on visual perception and experienced greater difficulty coordinating figural, symbolic, and discursive representations when solving geometric problems. In contrast, students at Level 1 demonstrated more systematic reasoning and were better able to relate visual representations to mathematical concepts, although differences remained in how male and female students expressed and justified their reasoning. Overall, female students showed greater flexibility in coordinating multiple representation strategies, whereas male students tended to rely more heavily on intuitive visualization. These findings provide the basis for interpreting students' geometric thinking through Duval's theory of semiotic representation.

Analysis Based on Van Hiele's Theory

The findings from the four research subjects reinforce the validity of the hierarchical nature of cognitive development in Van Hiele's Theory. There is a stark cognitive divide between the group of students at the visualization level (Level 0), represented by SL0 and SP0, and the group of students at the analysis level (Level 1), represented by SL1 and SP1.

Students at the visualization level tend to think in ways that are strongly influenced by the static, prototypical appearance of geometric objects. Their mathematical reasoning is often constrained when the spatial orientation of objects is manipulated, as changes in visual configuration make it more difficult for them to construct appropriate geometric representations. Students at this level generally perceive three-dimensional objects based on their overall appearance rather than identifying the constituent geometric components independently (Rahmah, 2021). Consequently, they often experience greater difficulty when solving problems involving non-prototypical geometric representations commonly absent from textbook examples. Similar findings were reported by Nopriana et al. (2023), who found that prospective mathematics teachers in Indonesia predominantly remained at the levels of informal analysis and deduction, suggesting that progression through the Van Hiele hierarchy continues to be a challenge across educational levels.

Students who have reached the analysis level demonstrate independent thinking that transcends the visual limitations of the surface. SL1 and SP1 are able to deconstruct three-dimensional shapes into a network of functional components governed by standard geometric laws. Analytical ability is confirmed by the mastery of the principle of spatial invariance. Students at the analysis level no longer view geometric figures as rigid, single entities, but rather as a set of interrelated, constant, and decomposable mathematical properties (Rahayu & Jupri, 2021). The VHGT, as a tool for measuring the Van Hiele hierarchy, has historically proven to be reliable and is consistently used across countries to identify shifts in students' cognitive levels of geometry (Senk et al., 2022).

Analysis Based on Duval's Theory

A student’s success or failure in solving geometry problems is not determined solely by their spatial intuition, but rather by their ability to effectively use different types of representational registers and accurately perform semiotic transformations (Duval, 2017). Students at the analysis level (SL1 and SP1) are able to optimize multifunctional figural registers to form accurate new spatial sketches, as well as formulate mathematically logical oral arguments. Visualization-level students (SL0 and SP0) often get caught up in the ambiguities of natural language mixed with non-mathematical physical perceptions. The students’ greatest difficulty in large-scale assessments lies precisely in the task of converting between different semiotic registers, not in processing within a single register (Ferretti et al., 2024).

Conversion transformations appeared to pose greater cognitive demands in the solid geometry tasks examined in this study. Among the participants, SP1 demonstrated greater flexibility in performing multidirectional conversion transformations on Problem-Solving Task 3 by successfully coordinating multiple semiotic representations. According to Duval (2006), mastery of conversion transformations between different semiotic registers is an important indicator of deep conceptual understanding in mathematics. In contrast, SL0 experienced considerable difficulty when converting textual descriptions into graphical representations and symbolic mathematical expressions. These findings support Duval (2006) argument that conversion is often more cognitively demanding than treatment because it requires learners to coordinate meaning across heterogeneous semiotic systems rather than merely applying procedural operations within a single representation system.

A study involving 881 high school students in Cyprus found that perceptual apprehension is significantly easier to develop than discursive apprehension, and that the relationships among the different types of apprehension vary across grade levels (Michael-Chrysanthou et al., 2024). These findings are consistent with the results of the present study, in which all participants demonstrated perceptual apprehension, whereas only SL1 and SP1 were able to effectively coordinate perceptual, discursive, and operative apprehension. Antunović & Manfreda Kolar (2026) argued that progress in learning geometry depends on the coordinated use of all four types of apprehension rather than on a linear developmental sequence. This perspective helps explain why SL0 experienced substantial difficulties in solving the geometric tasks despite demonstrating perceptual apprehension.

The findings of this study suggest that integrating Van Hiele's theory with Duval's theory provides a more comprehensive explanation of students' geometric thinking than using either framework independently. While Van Hiele's theory describes the developmental level of geometric reasoning, Duval's theory explains how that reasoning is expressed through the coordination of different semiotic representations. The present findings indicate that students at the same Van Hiele level may still differ in their ability to transform and coordinate representations. Therefore, combining both theoretical perspectives offers a more detailed understanding of students' strengths and difficulties in solving geometry problems.

Characteristics of the Thinking Process as Viewed from a Gender Perspective

The integration of Van Hiele's theory and Duval's theory in this study provides an integrated perspective for interpreting students' geometric thinking. Van Hiele's levels of geometric development describe the progression of students' geometric reasoning, whereas Duval's semiotic representation registers explain how this reasoning is expressed and coordinated through different forms of representation.

The group of female students exhibited a tendency toward formal-procedural thinking characteristics, perseverance, attention to detail regarding aspects of mathematical sequential naming, and strength in the verbal articulation of semiotics across multifunctional registers. SP1 demonstrated superior ability to flexibly control semiotic representations and present two alternatives for performing decimal calculations in a coherent and precise manner. Female students tend to excel in procedural accuracy in algebra and in the formal, structured organization of mathematical work (Putri & Nurlaelah, 2022). Female teacher candidates demonstrate a higher and more structured level of geometric thinking in mathematical discourse compared to their male counterparts (Nopriana et al., 2023).

The group of male students exhibited a tendency toward intuitive and global thinking, relying heavily on direct mental modeling, yet were prone to difficulties with formal discursive representation. SL1 quickly grasped the spatial ratio of water volume through his spatial intuition, but experienced verbal cognitive block when asked to explain the formal geometric theory behind it. Fifth- and eighth-grade students in Australia found that although boys excel at mental rotation, the unique contribution of spatial ability to mathematical performance varies depending on the subject matter and does not always benefit boys across the board (Harris et al., 2021). The SL1's intuitive spatial strengths do not automatically lead to superior discursive reasoning. The danger of male students' perceptual apprehension bias is that they jump directly to global visual conclusions without synchronizing them with formal discursive reasoning (Duval, 2017).

From a pedagogical perspective, these findings suggest that geometry instruction should not only emphasize students' progression through the Van Hiele levels but also provide learning experiences that explicitly develop the coordination of verbal, visual, and symbolic representations. Teachers can facilitate this process through tasks involving multiple representations, guided mathematical discussions, and dynamic geometry activities that encourage students to explain, visualize, and justify their reasoning. Such learning experiences may help students overcome representational difficulties and strengthen their geometric reasoning.

CONCLUSION

This study concludes that students' geometric thinking differs not only according to their Van Hiele level but also according to their ability to coordinate semiotic representations. Students at the Visualization level tended to rely on perceptual features and experienced greater difficulty transforming information between verbal, visual, and symbolic representations, whereas students at the Analysis level demonstrated more effective coordination of these representations, with the female participants in this study showing greater representational flexibility than the male participants. These findings suggest that integrating Van Hiele's theory with Duval's theory provides a more comprehensive framework for understanding students' geometric thinking by explaining both the developmental progression of geometric reasoning and the representational processes involved in problem solving. From a practical perspective, geometry instruction should incorporate learning activities that explicitly promote the coordination of multiple representations rather than focusing solely on conceptual understanding. Nevertheless, this study was limited by the small number of participants and its focus on a single geometry topic. Therefore, future research should involve more diverse participants and broader geometry topics to further examine the applicability of the integrated theoretical framework in different educational contexts.

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INFORMED CONSENT

The authors have obtained informed consent from all participants.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest.

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